Equilibrium Credit Spreads and the Macroeconomy*

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April 2014

Under Revision

ABSTRACT

The Great Recession of 2008-09 offers a primary example of the importance of credit risk to the macroeconomy. This paper develops a novel framework that brings together core ideas from asset pricing, capital structure, and macroeconomics within a tractable general equilibrium model with heterogeneous firms making optimal investment and financing decisions under uncertainty. Because credit risk premia are an important component of the cost of capital, movements in bond markets are propagated into the real economy generating a strong correlation between spreads and macro aggregates and producing large and skewed business cycles, with sharper, more pronounced, recessions.

*We are grateful to comments from Andy Abel, Hui Chen, Urban Jermann, Aubhik Kahn, Dmitry Livdan, Monika Piazzesi (editor) Neng Wang, three anonymous referees and participants at seminars in Calgary, Carnegie-Mellon, Duke, Wharton, NY Fed, Richmond Fed, ECB, UBC, UCLA, and Imperial College, as well as the following conferences: AFA, SED, Minnesota Macro Week, NBER Capital Markets, NYU Five Star, Rimini Macro-Finance, UCSB-LAEF, Venice C.R.E.D.I.T, and ESSFM Gerzensee. All errors remain our own.

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1 Introduction

The Great Recession of 2008-09 offers a primary example of the important role that fluctuations in credit risk play in the aggregate economy. Unfortunately these developments also exposed the current need for new state of the art models suitable to understand the joint behavior of credit risk, financial prices, and the key macroeconomic aggregates.

Attempts to provide an integrated discussion of these issues in a modern setting have gathered speed recently but most papers avoid some essential features of the data. Many general equilibrium models used in macroeconomics attempt to explain the cyclical behavior of credit markets quantities and their correlation with macroeconomic aggregates but largely abstract from variations in risk premia and asset prices.[1] By contrast, macro models that exploit the role of asset prices and risk premia explicitly, generally ignore the role of credit markets entirely.[2] Parallel efforts by financial economists have instead focused on explaining the magnitude of credit risk by linking it, initially, with the financing decisions of firms and, more recently, with exogenous movements in risk premia and aggregate factors.[3]

This paper offers a novel framework that brings together many of the core ideas from asset pricing, capital structure, and macroeconomics within a tractable general equilibrium model with heterogeneous firms that make optimal investment and financing decisions under uncertainty. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and required to be consistent with consumption and savings decisions of a representative household/investor. By integrating corporate investment and capital structure decisions into a modern asset pricing model, we endogenously link movements in aggregate quantities such as investment and output to plausible movements in the prices of stocks and bonds.

The joint endogeneity of financial prices and macro quantities implies that both the intertemporal elasticity of substitution and risk aversion matter separately for aggregate quantities, a result

[1] Classic examples include Carlstrom and Fuerst (1997), Kyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). Some recent examples are Lorenzoni and Walentin (2007) and Philippon (2008).

[2] Some examples are Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fischer (2001), Uhlig (2006), Guvenen (2009) and Loechster and Kaltenbrunner (2010).

[3] Building on Leland (1994) recent quantitatively successful contributions include Hackbarth, Miao, Morellec (2006), Chen, Collin Dufresne, Goldstein (2008), Chen (2008), Bhamra, Kuhn, and Strebualev (2008).

that contrasts strongly with extant macro literature where the role of risk aversion and thus risk premia is largely ignored.[4] As a result we can generate plausible movements in credit prices that are largely driven by fluctuations in risk premia and not by changes in average default rates alone.

Because credit spreads are an important component of the cost of capital for firms, risk premia in corporate bond markets are propagated into the real economy so that our model generates the strong correlation between credit spreads and macro aggregates observed in the data. Thus, the credit risk premium emerges as a common link between credit, equity markets and macroeconomic aggregates and its movements provide a novel and powerful amplification mechanism for macroeconomic fluctuations.[5]

While qualitatively we are able to replicate all the predicted cyclical patterns in macro and financial variables our parsimonious model also performs well quantitatively. We show that we can match both the level and volatility of key macro aggregates as well as that of equity and bond returns with realistic leverage and default levels. Economic fluctuations become amplified and also asymmetric with sharper recessions associated with pronounced spikes in both default rates and credit spreads.

In our model credit spreads predict output and investment growth because they capture important information about the cross-sectional distribution of firms as well as independent shocks to credit supply and not just because they are correlated with aggregate productivity. This suggests that credit spreads may also be of independent use for policy makers seeking to stabilize the economy.[6]

Recently other authors have tried to embed more realistic credit markets into aggregate equilibrium models. Jermann and Quadrini (2010) show that credit shocks are necessary to generate the observed behavior of US economy. Their model is significantly different in that a firm can

[4] An elegant example is Tallarini's (2000) separation result where risk aversion governs asset prices alone while the intertemporal elasticity of substitution determines economic aggregates.

[5] Examples of the ability of credit spreads to forecast economic activity include studies by Stock and Watson (1991), Lettau and Ludvigson (2004), Gilchrist and Zakrajsek (2008), and Mueller (2008). Keim and Stambaugh (1986), Schwert (1989) and Fama and French (1992) study the link between credit spreads and equity markets. There is also a complementary literature documenting that asset prices more broadly have significant forecasting power for macroeconomic variables - see Ang, Piazzesi, Wei (2006) for a recent study and Stock and Watson (1999) for a classic contribution.

[6] Gomes and Schmid (2010) study the optimal response of monetary policy to variations in credit risk in a similar setting.

make intra period loans that are subject to stochastic collateral constraints and face adjustment costs to distributions. Brunnermeier and Sannikov (2010) build a continuous time macroeconomic model with a more detailed financial sector. As in other classic papers by Kyotaki and Moore (1997), Bernanke et al (1999) and Cooley et al (2004) the authors show how the equilibrium could be fragile in that large negative shocks can be amplified by endogenous variations in the price of capital goods. Compared to our paper here, both of these models focus on a different set of financial frictions and offer a more detailed institutional setting. On the other hand they both consider representative firm frameworks and largely abstract from variation in risk premia.[7]

Methodologically, our model is also related to recent quantitative analysis of various aspects of the interactions between frictions at the corporate level, asset prices and macroeconomic fluctuations and to that on equilibrium asset pricing with heterogeneous firms.[8]

The rest of the paper is organized as follows. Section 2 describes our general equilibrium model and some of its properties, while Section 3 discusses some of the issues associated with solving it numerically. A detailed discussion of our findings is provided in Section 4, before we conclude.

2 The Model

In this section we describe a general equilibrium with heterogeneous firms that are financed with both debt and equity. Firms produce a unique final good that can be used for both consumption an investment. They own, and can add to, their capital stock by taking advantage of stochastic investment opportunities. Debt is used because of its tax benefits and because equity issues are costly. Hence the capital structure reflects combines the key elements of both modern trade off and pecking order theories. Both debt and equity can be issued regularly although there are issuance costs. Excessive debt may cause some firms to default. On the other hand attractive business and credit conditions may also encourage new entrants to join in production.

[7]Still more recently Miao and Wang (2010) extend our framework to allow for endogenous labor supply, while Gourio (2010) introduces disaster risk in a simplified setting where firms are only alive for two periods. This ensures that firm heterogeneity plays no role in equilibrium and dramatically speeds up computation on the model.

[8]Recent examples of the former include Bolton, Chen, Wang (2010), Hugonnier, Malamud, Morellec (2010), Eberly, Rebelo and Vincent (2010), Eberly and Wang (2010), and Ai, Croce, Li (2010). For the latter see Berk, Green and Naik (1999) and Gomes, Kogan, and Zhang (2003).

We believe this model merges many key features surrounding the investment and financing behavior of firms in a modern asset pricing setting.

But our very detailed focus on credit and equity markets carries one cost with respect to other macro models in that we abstract entirely from labor market fluctuations. This is done only to maintain focus and preserve economic intuition. Adding a realistic labor market component that interacts meaningfully with the financial market conditions is a great step, best taken separately.[9]

2.1 Firms

The production sector of the economy is made of a continuum of firms that differ in their productivity, size and leverage among other characteristics. In describing the problem of firms we take the stochastic discount factor for the economy as given. We show later how this is determined in general equilibrium by the optimal consumption and savings decisions of households. Nevertheless it is important to recognize that a firm's discount rates depend on the aggregate state of the economy, denoted s. We will show below that this includes both the current state of the aggregate shocks and the equilibrium cross-sectional distribution of firms.

2.1.1 Technology

All firms produce the same homogeneous final good that can be used for consumption or investment. The production function denoting the instantaneous flow of output is described by the expression:

$$y_{jt} = \exp(x_t + z_{jt})k_{jt} \tag{1}$$

where k_{jt} denotes the firm's productive capacity and x_t and z_{jt} denote the values of aggregate and firm specific productivity, respectively. The behavior of these follows a first order autoregressive

[9] Some recent papers include both credit frictions and labor markets, including for example Bernanke et al (1999), Jermann and Quadrini (2010) and, more recently, Khan and Thomas (2013). But none seeks to match the movements in equity and credit prices as well. Gilchrist et all (2011) matches credit spreads and macro quantities but is not interested in equity prices or the capital structure. On the other hand Gourio (2011) matches financial prices as well as we do but uses what is effectively a single firm model.

process with normal innovations:

$$x_t = (1-\rho_x)\bar{x} + \rho_x x_{t-1} + \sigma_x v_{xt} \tag{2}$$

$$z_{jt} = \rho_z z_{j,t-1} + \sigma_z v_{zjt} \tag{3}$$

where v_{xt} and v_{zjt} are independently and identically distributed shock drawn from standard normal distributions. We use $N(x_{t+1}|x_t)$ and $N(z_{t+1}|z_t)$ to denote the conditional cumulative c.d.f of these two variables.

A growing literature has emphasized the importance of non-normal or disaster shocks and time variation in volatility (e.g. Bloom (2009), Gourio (2010), Gilchrist et al (2011)). Although we could easily include these features as well, we choose not to include them to illustrate better how a general equilibrium model can generate endogenous stochastic consumption volatility that is a feature of several popular asset pricing models with exogenous consumption (e.g Drechsler and Yaron (2010)).

2.1.2 Investment Opportunities

Each firm's existing stock of capital, k_{jt}, is assumed to depreciate at the periodic rate of δ. However in each period firms also have the opportunity to invest and increase next period's stock of capital $k_{j,t+1}$. Investment takes place by adopting a new project of discrete size. Each adopted project costs, i, goods per unit of capital, and it scales the stock of capital to $k_{j,t+1} = g \times k_{jt}$. In other words, to increase next period's stock of capital by a (net) factor of $g-1$ the firm must surrender $i.k_{jt}$ units of current cash flow.

The usual assumption is of course that $i = g - 1$ at all times. Here we generalize it to allow for the investment cost to be stochastic and differ across firms. As a result only firms drawing investment costs below a cutoff value, $\bar{i}_t$, will choose to increase their productive capacity. This in turn will lead to an endogenous cross-sectional variation in firm size over time as firms optimally take advantage of differing investment opportunities.

Formally, we assume the cumulative distribution of investment costs, denoted $H(i)$, is uniform and independent over time. We further set $\mathrm{E}i = g - 1$ and write the law of motion for a typical firm's exiting stock of capital, k_t, as:

$$k_{j,t+1} = \begin{cases} (1-\delta)k_{jt} & \text{with prob.} \quad 1 - H(\bar{i}_t) \\ gk_{jt} & \text{with prob.} \quad H(\bar{i}_t) \end{cases} \tag{4}$$

2.1.3 Financing

Firm's can finance part of their spending thorough debt. We assume that this takes the form of a callable consol bond that pays a fixed coupon $b_{jt}.k_{jt}$ as long as the debt is not called or the firm does not default on its obligations.[10]

New debt can be issued in every period. To avoid dealing with multiple state variables at the same time however we assume that all existing debt is recalled at the same time. Without loss of generality we assume that debt is always recalled at market value, denoted $B(k, b, z, s)$. In words, this is the current value of a callable, defaultable, claim on a firm of size $k = k_{jt}$ with current idiosyncratic productivity $z = z_{jt}$, that promises to pay $b.k = b_{jt}.k_{jt}$ per period, at a time when the aggregate state of the economy is given by $s = s_t$.

It follows that, except for gross investment expenditures, the after-tax cash flows to the firm's equity holders, $\Pi(\cdot)$ are given by:

$$\Pi(k, b, b', z, s) = (1-\tau)(\exp(x+z) - b - \delta)k - B(k, b', z, s) - (1 + \chi_b \kappa_b)B(k, b, z, s) \tag{5}$$

where we use the notation $b' = b_{j,t+1}$ and the indicator function χ_b takes the value of 1 when the firm changes its debt decision, i.e. $b \neq b'$. The variable $\kappa_b \geq 0$ in equation captures transaction costs, such as underwriting fees, associated with calling and reissuing debt, while τ denotes the effective tax rate on profits adjusted for taxes on distributions and personal interest income.

Linearity of technology, investment and default costs in k allows us to define and work with the normalized equity value function $P(s, z, b) = V(s, z, b, k)/k$. Similarly, we will use $Q(s, z, b) = B(s, z, b, k)/k$ denote the normalized market value of debt - effectively a measure of book leverage.

[10] Defining the coupon as $b_{jt}k_{jt}$ allows us to interpret b_{jt} as a measure of book leverage for the firm.

Finally, firms can also fund themselves with new equity issues. Equity issues too are costly and we use $\kappa_e \geq 0$ to capture the unit costs associated with issuing any new equity.

2.1.4 Default and Debt Pricing

As discussed above, bondholders receive a coupon payment bk as long as the firm does not default or recalled. If debt is called they pocket the current market value of the debt, $B(k, b, z, s)$. The only scenario under which they experience losses is upon default. Limited liability ensures that its is optimal for equity holders to default on their debt obligations whenever the (normalized) value of equity, $P(b, z, s)$, becomes negative. Mathematically, this yields a default cutoff value for the idiosyncratic shock, $\bar{z}(b, x)$, such that the firm will default whenever $z < \bar{z}(b, s)$. Formally:

$$\bar{z}(b, s) = \min\{z : P(b, z, s) = 0\} \tag{6}$$

We show below that $\bar{z}$ is increasing in leverage b and declining in aggregate productivity x.

If default occurs we assume that the firm's assets (its capital plus current cash flows) are liquidated and the proceeds used to pay its creditors. A fraction $\phi > 0$ of these assets however is lost in liquidation so that creditors recover an amount equal to $(1-\phi)(1-\delta+xz)k$. Given these possibilities, the normalized market value, $B(k, b', z, s)$, of a claim promising to pay b' tomorrow, in a firm currently in state (k, b, z, s), obeys the recursion:

$$\begin{aligned} B(k, b', z, s) \quad &= \quad \mathrm{E}M(s, s') \left[\int_{\bar{z}(b', s')} [b' + B(k', b', z', s')] dN(z'|z) \right. \\ &+ \quad \left. \int^{\bar{z}(b', s')} (1-\phi)(1-\delta+\exp(x'+z'))k' dN(z'|z) \right] \end{aligned} \tag{7}$$

where we take households/investors stochastic discount factor, $M(s, s')$, as given for the moment. Thus, this value depends on firm specific characteristics, (k, z), and the aggregate state of the economy, $s = (x, \mu)$. The following Lemma establishes the key general properties of the market value of debt.

Lemma 1. *The market value of debt, $B(\cdot)$, is increasing in x and z.*

Proof Monotonicity in x follows from the facts that $\bar{z}(\cdot)$ is decreasing and the recovery payment increasing, in x', plus the persistence in (2). Monotonicity in z follows from the persistence in (3) and the fact that the recovery payment also increasing in z'. □

As in Jermann and Quadrini (2011) and Kahn and Thomas (2013) we also consider a version of the model with credit market shocks. To do this we assume that recovery rates in bankruptcy, ϕ, fluctuate over time, perhaps as a result of shocks to liquidation values or "liquidity". Formally, let $\Gamma(\phi'|\phi)$ denote the conditional distribution of expected liquidity shocks next period.[11,12]

Equation (7) shows how fluctuations in the recovery rate, ϕ, directly affect the relative price of credit to the firm. Thus changes in ϕ act as effective shocks to credit supply, leading to tighter credit conditions and increases in credit spreads - a very natural form of introducing financial shocks in our model.

For the purposes of comparison to the data we also define the creditor's recovery upon default:

$$\frac{(1-\phi)(1-\delta+\exp(x+z))}{Q^0(b)} \tag{8}$$

where $Q^0(b)$ is the (normalized) value of the debt upon initial issuance. Similarly, it is also useful to define the yield $y(z,b,s)$ on a risky corporate debt issue as:

$$y(z,b,s) = b/Q(b,z,s) \tag{9}$$

2.1.5 Equity Value and Optimal Policies

We can now characterize the decisions of equity holders in detail. At every point in time the equity value (per unit of capital) obeys:

$$P(b,z,s) = \max\{P^0(b,z,s), P^I(b,z,s)\} \tag{10}$$

[11] Einsfelt and Rampini (2007) show these types of shocks can be important to explain measured variation in individual firm investment over time.

[12] In general $\Gamma(\cdot)$ could also exhibit dependence on aggregate productivity shocks x. We ignore this possibility here to better understand the independent effects of financial shocks.

where $P^I(b,z,s)$ denotes the equity value of a firm *after* it adjusts its stock of capital and $P^0(b,z,s)$ denotes that of a firm that chooses not to invest at all. The inaction value, $P^0(\cdot)$ is determined recursively by the Bellman equation:

$$P^0(b,z,s) = \max\{0, \max_{b'}\{\chi_e(1+\kappa_e)\pi(b,b',z,s) + EM(s,s')\int_{\bar{z}(b',s')} P(b',z',s')N(dz'|z)\}\} \quad (11)$$

Here $\pi(\cdot) = \Pi(\cdot)/k$ and χ_e is an indicator function that takes the value of 1 when the firm raises new equity and pays issuance costs $\kappa_e \geq 0$. The truncation in the continuation value reflects the impact of the possibility of default on the returns to equity holders. In turn the value of investing, $P^I(\cdot)$ obeys:

$$\begin{aligned} P^I(b,z,s) &= \max\{0, \max_{b'}\{\chi_e(1+\kappa_e)\left[(1-\tau)(\exp(x+z)-b-\delta)-i\right. \quad (12) \\ &- \left.(1+\chi_b\kappa_b)Q(b,z,s) + g.Q(b',z,s)\right] + g.EM(s,s')\int_{\bar{z}(b',s')} P(b',z',s')N(dz'|z)\}\} \end{aligned}$$

It follows that there is an optimal investment cutoff that then obtained from:

$$\bar{i}(b,z,s) = (g-1)\left[\max_{b'}\left\{\frac{\mathrm{E}M'\int_{\bar{z}'} P(b',z',s')dz}{\chi_e(1+\kappa_e)} + Q(b',z,s)\right\}\right] \quad (13)$$

When equity issuance costs, κ_e, are 0, the term in square brackets is exactly Tobin's average q. It equals the expected value of all equity and debt claims on the firm, normalized by the value of the current stock of capital. In this case the optimal investment rule is imply that a firm will invest if and only if Tobin's q exceeds $i/(g-1)$. For the marginal firm this is exactly 1, so that at the aggregate level this economy behaves very much like one with an aggregate investment technology exhibiting convex adjustment costs.

2.1.6 Properties of the Firm's Problem

All individual firm decisions depend on the aggregate state of the economy, s, which includes the current state of aggregate productivity x. Although much of our analysis is quantitative we can nevertheless establish a number of useful general properties about the value and policy functions above.

Lemma 2. *The normalized equity value $P(b, z, s)$, is increasing in x and z, and declining in leverage b;*

Proof This follows directly from the fact that equity cash flows, $\pi(b, b', z, s)$, net of investment spending, i, are increasing in x and z and declining in b. $\square$

Monotonicity of the value function is unsurprising but crucial to ensure the existence of a (unique) default threshold. In addition to these properties, we can show that limited liability which endows equity with an exit option and increases the value of uncertainty, implies that $P(\cdot)$ will be convex in $\exp(x)$ and $\exp(z)$.

Lemma 3. *The default cutoff, $\bar{z}(b, s)$, is decreasing in x and increasing in the coupon b.*

Proof This follows from the fact that $P(\cdot)$ is increasing in x and declining in b.$\square$

It can also be showed that if the discount factor $M(\cdot)$ is constant the default cut-off $\bar{z}(b, s)$ would be linear in x. In this case recessions and expansions in aggregate productivity will produce symmetric responses in the the default cutoff. As a result, log-linearized models with uniform distributions for idiosyncratic shocks, z, counterfactually imply symmetric fluctuations in firm default rates over the business cycle. Introducing risk premia, by allowing $M(\cdot)$ to be be tied endogenously to x as well as a log-normal distribution for shocks z allows us to obtain asymmetric responses to aggregate shocks.[13]

Lemma 4. *The investment cutoff, $\bar{i}(b, z, s)$, is increasing in x and z and (weakly) decreasing in the existing coupon payment b. The optimal debt policy $\bar{b}(z, s)$ is increasing in x and z.*

Proof The monotonicity of the policy functions in x and z follows directly from Lemmas 1 and 2. In addition, higher b might trigger the equity issuance indicator $\chi_e > 0$ to switch from 0 to 1, reducing $\bar{i}(\cdot)$ implied by (13). $\square$

[13] Some examples are Bernanke et al (1997), Gertler and Karadi (2010).

Many of these properties are relatively intuitive. However, the fact that the investment cutoff $\bar{i}(\cdot)$ is declining in the existing coupon payment b, means that our model delivers a form of "debt overhang" result. High leverage firms are less likely to invest.

Figure **??** illustrates the properties of the (normalized) equity and debt value functions, $P(\cdot)$ and $Q(\cdot)$. Figure **??** shows the key properties of the optimal default, investment and debt policies using the benchmark parameter values discussed below.

2.2 Aggregation

To characterize the general equilibrium of the model we must aggregate the optimal policies of each individual firm to construct macroeconomic quantities for our economy.

2.2.1 Cross-Sectional Distribution of Firm

We begin by defining $\mu_t = \mu(b, z, x, \phi)$ as the cross-sectional distribution of firms over leverage, b, and idiosyncratic productivity, z, at the beginning of period t, when the state of aggregate productivity is x and the recovery rate on assets in default ϕ.

Our timing is chosen so that that $\mu(\cdot)$ is constructed before any current period decisions take place. As is well known this cross-sectional distribution will move over time in response to the aggregate state of the economy and this is the main computational obstacle to solving the model.

In what follows it is useful to define the total mass of firms at the beginning of the current period as:

$$F(s) = F_t = \int d\mu_t \tag{14}$$

Like $\mu(\cdot)$ itself, $F(s)$ is constructed before individual firms' decisions are made.

Similarly, we can construct the equilibrium default rate in the economy as:

$$D(s) = 1 - \frac{\int_{z \geq \bar{z}(b,s)} d\mu}{F(s)} \tag{15}$$

Since the default threshold, $\bar{z}(b, s)$, is decreasing in x this default rate will be countercyclical and, as discussed above, will generally respond asymmetrically to positive and negative shocks in x.

2.2.2 Firm Entry

Entry is necessary in the model to replace bankrupt firms and ensure a stationary distribution of firms in equilibrium. Accordingly, we assume that every period a mass of potential new entrants arrives in the economy. Potential entrants behave similarly to incumbents but face different initial conditions. Specifically, potential new entrants:

- have no initial capital, so that $k_{jt} = 0$, and thus begin production one period after entry;
- have no initial level of debt, so that $b_{jt} = 0$
- draw an initial realization of the idiosyncratic shock, $z_{j,t+1}$, from the long-run invariant distribution implied by (3), denoted $N^{\star}(z')$;

Like incumbents, potential entrants choose their optimal level of debt and investment. Since they start with 0 capital, we assume that potential entrants must make an initial investment of size $\alpha \bar{k}_t$, where:

$$\bar{k}_t = \frac{\int k_{jt} d\mu_t}{F_t} \tag{16}$$

denotes average firm size at time t. We will assume new firms start small so that $\alpha < 1$. We can also think of this initial investment as the cost of entry in the market.

Like incumbents, entrants differ in the cost of this initial investment. For the sake of symmetry and parsimony we assume that the unit cost of their investment opportunities, e is also drawn from the c.d.f. $H(e)$. This implies that only firms drawing a entry cost below a cutoff, $\bar{e}(z, s)$, find it optimal to invest and thus enter the market.

2.2.3 Aggregate Investment

Given the optimal behavior of individual firms, gross aggregate investment is equal to:

$$I(s) \;\; = \;\; \int_0^{\bar{i}(s)} ikdH(i) + \int_0^{\bar{e}(s)} e\bar{k}dH(e) - \int (1-\chi)k \, d\mu + \int \delta k \, d\mu \tag{17}$$

The first two terms capture the total sum of investment costs for the newly adopted projects, by all existing firms and by new entrants respectively. Next, we net out the disinvestment associated with asset liquidation by exiting firms. The last term then adds the depreciation expenditures of all existing firms, at the beginning of the period.

Since each firm's investment and default decisions are both independent of their current stock of capital, k, the law of large numbers implies that

$$\begin{aligned} I(s) &= \bar{k}\left[\int_0^{\bar{i}(s)} idH(i) + \int_0^{\bar{e}(s)} edH(e) - \int(1-\chi)d\mu\right] + \delta\int kd\mu \\ &= K(s)\left[\int_0^{\bar{i}(s)} idH(i)/F(s) + \int_0^{\bar{e}(s)} edH(e)/F(s) - D(s) + \delta\right] \end{aligned} \tag{18}$$

where $K(s) = \int kd\mu(s) = \bar{k}(s)F(s)$ is the aggregate capital stock in the economy, when the aggregate state is s.

Together with our assumptions regarding the linearity of the aggregate production technology, this expression ensures that the model economy will grow endogenously over time at a stochastic rate that is linked to average aggregate productivity x_t. Faced with aggregate shocks our economy will exhibit persistent variation over time in the growth rates of output and consumption among others. This provides a natural laboratory to investigate the effects of sorts of shocks to long run growth rates in a general equilibrium context with endogenous quantities and prices.

Note also that, if the arrival of investment projects to new and old firms $H(\cdot)$ is time varying, the model easily accommodates the type of investment specific technological shocks that have been emphasized recently in the literature.[14]

Finally, the expression for aggregate investment (17) integrates elements of rising marginal adjustment costs and partial irreversibility, both of which are important to generate quantitatively interesting behavior in asset prices. Since the optimal investment cutoffs are increasing in productivity, the marginal cost of (aggregate) investment rises in good times, much like it would in a simple aggregate model with standard convex adjustment costs (e.g. Jermann (1998)). And because bankruptcy is costly, investment is, effectively, only partially reversible. As default is more likely in recessions, this feature adds cyclical variation to consumption growth in general equilibrium.

[14] e.g. Greenwood, Hercowitz and Huffman (1988), Fisher (2006), Cummins and Violante (2007), Papanikolaou (2011) and Kogan and Papanikolaou (2013)

This endogenously increases the market price of risk during recessions and exacerbates underlying variations in equilibrium asset prices.

2.2.4 Other Aggregate Quantities

Other aggregate quantities are relatively straightforward to define. Aggregate output can be defined as:

$$Y(s) = \int \exp(x+z)k \, d\mu = \exp(x)K(s) \tag{19}$$

and the losses associated with bankruptcy are given by:

$$\Phi(s) = \int (1-\chi)\phi(1+\exp(x+z))k \, d\mu \tag{20}$$

Finally we can also construct the aggregate market value of corporate equity and debt respectively with the expressions:

$$V(s) = \int Q(s,z,b)k \, d\mu \tag{21}$$

and

$$B(s) = \int B(s,z,b)k \, d\mu \tag{22}$$

These definitions for the aggregate quantities make it clear that the aggregate state of our economy s is the triplet (x, ϕ, μ). All aggregate quantities and prices depend on the average state of productivity, financial conditions as well as the cross-sectional variation in firm productivities and leverage.

2.3 Households

To close our general equilibrium model we now describe the behavior and constraints faced by the households/investors. These are now fairly standard. We assume that our economy is populated by a competitive representative agent household, that derives utility from the consumption flow of the single consumption good, C_t. This representative household maximizes the discounted value of future utility flows, defined through the Epstein-Zin (1991) and Weil (1990) recursive function:

$$U_t = \{(1-\beta)u(C_t)^{1-1/\sigma} + \beta\,\mathrm{E}_t[U_{t+1}^{1-\gamma}]^{1/\kappa}\}^{1/(1-1/\sigma)}. \tag{23}$$

The parameter $\beta \in (0, 1)$ is the household's subjective discount factor and $\gamma > 0$ is the coefficient of relative risk aversion. The parameter $\sigma \geq 0$ denotes the elasticity of intertemporal substitution and $\kappa = (1-\gamma)/(1-1/\sigma)$.

Our household invests in shares of each existing firm as well as riskless bond which in zero net supply and earns a period rate of interest r_t. We also assume that there are no constraints on short sales or borrowing and that households receive the proceeds of corporate income taxes as a lump-sum rebate equal to:

$$T(s) = \tau \int \exp(x+z)k\,d\mu = \tau \exp(x)K(s) \tag{24}$$

Given these assumptions the equilibrium stochastic discount factor for our economy between two adjacent periods is defined by the expression:

$$M_{t,t+1} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma} R_{W,t+1}^{1-1/\kappa}\right]^{\kappa}. \tag{25}$$

where

$$R_{W,t+1} = \frac{W_{t+1} + C_{t+1}}{W_t}. \tag{26}$$

is the return on total household wealth, including bonds and tax proceeds.

As is well known, the absence of arbitrage implies that all gross asset returns in this economy will satisfy:

$$\mathrm{E}_t[M_{t+1}R_{i,t+1}] = 1, \tag{27}$$

for all assets i.

2.4 General Equilibrium

We have shown how investor behavior determines the equilibrium stochastic discount factor, $M_{t,t+s}$, given household wealth. Earlier we described optimal firm behavior given the stochastic discount factor and shown how it determines aggregate investment and output as well as household wealth. Ensuring consistency between these two pieces of the economy requires that aggregate consumption by households is equal to aggregate production, net of investment and deadweight losses.

Formally our competitive equilibrium can then be constructed by imposing the additional consistency condition:

$$C_t = C(s) = Y(s) - I(s) - \Phi(s) \tag{28}$$

This ensures that the stochastic discount factor used by each firm corresponds to that implied by optimal household behavior.[15]

Finally we also need to specify a law of motion for the cross-sectional measure of firms over time. Given optimal firm policies this measure satisfies the following relation:

$$\begin{aligned} \mu(z', b', x', \phi') &= Prob(z_{t+1} < z', b_{t+1} < b', x_{t+1} < x', \phi_{t+1} < \phi') \qquad (29) \\ &= \Gamma(\phi'|\phi) N(x'|x) \left[\int \chi N(z'|z) \Omega_{b(z,b,x,\phi)=b'} d\mu(z,b,x,\phi) + N^\star \left(z'\right) \Omega_{b(0,0,x,\phi)=b'} \right] \end{aligned}$$

where Ω is an indicator function that takes the value of 1 if the optimal policy function $b(z, b, x, \phi)$ equals b' and 0 otherwise. $N(\cdot)$, $N^\star(\cdot)$ and $\Gamma(\cdot)$ are the cumulative distributions defined earlier.

The terms outside brackets in equation (29) capture the exogenous evolution in the aggregate states. The first term Inside the brackets sums all the surviving firms which choose optimal leverage b' across all current states next period. The second term adds the mass of all entering firms that also choose optimal leverage equal to b'. Recall that new firms arrive at the current period with $z = b = 0$.

Figure **??** illustrates the pattern of this cross-sectional distribution in when the technology shock is above and below its mean. Although much of the underlying variation is (log) normal

[15] We follow the convention of considering that bankruptcy costs are deadweight losses but in a general equilibrium model this is a somewhat debatable choice, since some of these costs might be in the form of legal and accounting fees that accrue to other types of firms in the economy.

the equilibrium distribution reflects both the effects of truncation by exit and lumpy additions from new entrants. We can see that there is generally less mass over high leverage, b, states in bad times, both because these firms are prone to default and also because potential entrants will typically choose lower leverage in recessions.

3 Computation and Calibration

We now offer a brief description of our approach to solve the model in section 2 and the choice of parameter values. As discussed above, the main obstacle to the computation of the competitive equilibrium is the fact that the cross-section measure of firms $\mu(\cdot)$ changes over time. In spite of this and the level of detail in capturing firm behavior, the model is remains relatively parsimonious and relies on relatively few independent parameters.

3.1 Computation

Computing the competitive equilibrium requires the following three basic steps:

- Given an initial stochastic discount factor $M_{t,t+s}$ solve the problem of each individual firms and determine the optimal level of entry and default

- Aggregate individual firm decisions and use the consistency condition (28) to compute aggregate consumption and wealth

- Ensure that the implied aggregate quantities are consistent with the initial process for $M_{t,t+s}$.

Convergence of this procedure delivers the equilibrium values for all individual and aggregate quantities in the model. Appendix 5 described this procedure in more detail.

3.2 Parameter Choices

3.2.1 Benchmark Model

We start with a benchmark model with no credit market shocks where the recovery rate, ϕ, does not move over time and there are no costs of issuing equity so $\lambda = 0$. This benchmark model requires us to specify the value of twelve parameters: three for preferences, eight for technology, and two more capturing institutional or policy features. Table 1 reports these choices.

The preference parameters are β, γ and σ. They are chosen to ensure that the model matches the key properties of the risk free rate and the aggregate equity premium in the economy. Several studies have already shown how to combine time non-separable preferences and persistent shocks to aggregate growth to produce these results. More recent papers have expanded this analysis to general equilibrium models with all equity firms. Our parameter values are quite similar to several papers in this literature.[16]

For the technology parameters we start by setting the depreciation rate of capital to 5% per year. The volatility and persistence of the aggregate productivity process are set to $\rho_x = 0.96$ and $\sigma_x = 0.015$. These values are largely in line with other macro studies and ensure that we match the volatility and persistence of output growth in the data. The parameters for idiosyncratic shocks determine the amount of cross-sectional variation in firm heterogeneity. Since we are especially concerned with the role of leverage and credit spreads in our economy we set these parameters to match the unconditional means of both of these variables. This implies that $\rho_z = 0.9$ and $\sigma_z = 0.18$

The parameter α governs the relative size of new entrants.

Finally we need the two institutional parameters. The marginal corporate tax rate, τ is set to 20% to reflect the effect of of individual taxes on distributions and interest on the effective marginal tax rate. We choose the bankruptcy cost parameter, ϕ to generate average recoveries on defaulted bonds around 75% of face value. This is close to the numbers reported in Warner (1977) and captures both direct and indirect costs of the bankruptcy process. Formally then we set the value of ϕ so that in default:

$$(1 - \phi)(1 - \delta + \exp(x + z)) = 0.75B(s_0, b, z) \tag{30}$$

where $B(s_0, b, z)$ is the value of debt (relative to k) initially raised by the firm on average.

[16] Early examples include Bansal and Yaron (2004) in models with exogenous quantities and Lochstoer and Kaltenbrunner (2009) and Croce (2010) in settings with production.

3.2.2 Additional Parameters

In addition to our baseline specification we also consider is the introduction of credit market specific shocks, along the lines of Jermann and Quadrini (2010). To do this we assume that recovery rates in bankruptcy fluctuate over time, perhaps as a result of shocks to liquidation values or "liquidity".[17] Equation (7) shows how fluctuations in the recovery rate, ϕ, directly affect the relative price of credit to the firm so that these liquidity shocks act as effective shocks to credit supply.

Formally we assume that ϕ can take two values: a benchmark value of 0.25 reflecting average bankruptcy costs and an extreme (but rare) value of 0.75 that occurs during liquidity crisis. We also assume that ϕ evolves over time according to a two-state Markov chain with the following transition probabilities:

$$P[\phi_{t+1} = 0.25|\phi_t = 0.25] = 0.98, \tag{31}$$

$$P[\phi_{t+1} = 0.75|\phi_t = 0.75] = 0.5, \tag{32}$$

In practice this means that liquidity crisis are both rare and temporary.

We also study a version with equity issuance costs, where the marginal cost of issue equity, $\lambda - 0.02$, an estimate that is similar to that used in Gomes (2001) and Hennessy and Whited (2006).

Finally, and for comparison purposes, we also study a simplified version of the model where we impose that the firm's initial investment must be entirely financed with equity.

4 Findings

We are now ready to describe our quantitative findings. We begin by summarizing the basic properties of the model by reporting the unconditional means and volatilities of the major aggregate quantities and asset prices. We then examine the model's implications for the behavior of financial variables over the business cycle and compare those with the available empirical evidence. Finally

[17] Einsfelt and Rampini (2007) show these types of shocks can be important to explain measured variation in individual firm investment over time.

we investigate the role of credit spreads in predicting future movements in both macro quantities.

To construct the statistics reported below we solve the model by numerical dynamic programming as detailed in Section 3. We then simulate the implied equilibrium policies at quarterly frequency to construct 1000 independent panels of 59 years each and report averages across all simulations. Unless otherwise noted we always report the relevant empirical moments for the sample period between 1951 and 2009.

4.1 Basic Properties

The first panel in Table 2 reports the volatility of the key macroeconomic variables as well as the share of investment in GDP for our benchmark model as well as several alternative parameterizations. The baseline parameter choices yield a very close match between the model and the data along these dimensions. Not only is the share of investment (and hence consumption) plausible but both variables also exhibit as much variability as in the actual data.

The lower panel in Table 2 documents the implied properties of the model for the unconditional means and volatilities of the risk free rate and the equity premium. Our benchmark model also does a good job in replicating these facts. Both the level of the risk free rate and the equity premium are very close to those observed in the data, and this match does not require the very large movements in the risk free rate often associated with habit preferences. This is because the persistent stochastic variation in growth rates generated by our model increases the household's precautionary savings thereby lowering equilibrium interest rates.

While Bansal and Yaron (2004) have shown that accounting for long run movements in consumption and dividends, combined with preferences for a early resolution of uncertainty, delivers realistic risk premia in an endowment economy setting, this has proved harder to implement in general equilibrium production economies (Kaltenbrunner and Lochstoer (2010), Campanale, Castro and Clementi (2009), Croce(2010)). This is because in a production economy, general equilibrium restrictions often tie dividends very closely to consumption, while empirically, dividends are much more volatile than consumption.

4.1.1 The Role of Leverage and Persistence

Here however financial leverage (endogenously) breaks the tight link between dividends and consumption and renders dividends an order of magnitude more volatile. This allows us to generate a more realistic amount of stock market volatility and is crucial in matching the aggregate equity premia.[18]

As Table 2 shows an all equity version of our economy does not generate enough volatility in equity returns and is not capable of matching the observed equity premium. The same occurs if the shocks to the stochastic growth rate are not sufficiently persistent. As we already discussed in this case the model also produces an unrealistically high equilibrium risk free rate. Persistence has profound effects on the asset pricing implications of our model but does not alter significantly the level or volatility of the main macro quantities, at least not at these relatively short horizons.

Predictably, financial leverage increases the volatilities of both financial prices and macro quantities. Compared with an all equity economy, unconditional volatility increases by about 25% to 30% in the baseline, levered, economy, which is roughly in line with the financial accelerator model in Bernanke et all (1999). However as we discuss in more detail below our underlying transmission mechanism is different and relies on movements in credit risk premia.

4.1.2 Investment Shocks

As Table 2 shows modifying the arrival rate of new projects affects the relative volatility of consumption and investment because it changes the marginal cost of allocating resources to the production of capital goods, i.e. creating new firms in the economy.

When $h'(x) = 0$ the arrival rate of new projects does not vary with business cycle conditions and the investment volatility is too low, while consumption volatility rises. This is remediated in our baseline case, because more good projects arrive in good times making it easier to transform consumption into capital. The result is that more firms are created and aggregate investment volatility raises. To see this formally, recall that investment arising from firm creation equals

[18] Although we do not report these numbers here the model also generates a slow moving pattern in leverage (Lemmon, Roberts and Zender (2008)) and the long run movements in aggregate dividends observed in the data (Bansal and Yaron (2004)).

$I = h\bar{e}^2 kF(s)$, where $F(s)$ is the mass of firms at the beginning of period t. If the behavior of the entry cutoff $\bar{e}$ remains unaffected, any increase in h increases average investment expenditures. In equilibrium however $\bar{e}$ actually becomes more cyclical when $h'(x) > 0$ reflecting the fact that investment becomes more attractive in good times.

General equilibrium implies that equity prices are also affected. One way to see this has to do with the implied movements in the endogenous stochastic discount factor since, in equilibrium, the cyclicality of investment prices, $\bar{e}$, is reflected both on expected consumption growth and on the returns to wealth. In turn this means that $h'(x) > 0$ makes equity -and corporate debt too- riskier.

Simplified versions of our baseline model without issuance costs for equity or that introduce credit shocks have only a negligible impact on these core macro and finance statistics. Their effect is instead felt on the credit market statistics and the conditional responses of economy to various shocks.

4.2 Credit Market Statistics

Table 3 shows the basic properties of the key credit market statistics as well as its empirical counterparts. Note that these statistics are all based on the average properties of the cross-sectional distribution of firms. As before we report the numbers both for our baseline parameter choices and for a few alternative specifications to illustrate some of mechanisms behind our findings.

The benchmark model almost exactly matches the cross-sectional average market leverage (the ratio of book leverage to the value of market equity plus book leverage). Moreover it also produces a realistic level for credit defaults and the average credit spread.[19]

4.2.1 The Role of Credit Risk

The combination of low default rates and substantial tax benefits to debt is often interpreted as evidence that firms chose sub-optimal levels of leverage in the data. Here we can easily match the observed leverage ratios for two reasons. The first is that our model produces a large credit risk

[19]By construction we only focus on spreads for long maturity debt. The existence of large spreads at short maturities is arguably a larger puzzle in the literature. Addressing this requires allowing for multiple forms of debt and probably more complex stochastic processes.

premium. As in recent work by Bhamra et al (2008) and Chen (2008), we exploit the fact that default occurs in periods of very high marginal utility, thereby significantly increasing the effective cost of default and the required compensation to bondholders. Here, however, this link is an explicit product of our general equilibrium structure and not an exogenously assumed covariance structure between default and the market price of risk.

Tying macroeconomic fluctuations to variation in default rates is the key component of the large credit spreads and the reason we can match the data along this dimension. Our baseline model generates a credit spread of 104 basis points with a default rate of only 1.42% while a risk neutral valuation would imply a credit spread of, at most 36 basis points.[20] Unlike other macro models with credit markets, it is then the credit risk premium, induced by the (endogenous) covariance between default rates and the market price of risk, and not default rates that account for the large credit spreads in our model.[21]

4.2.2 Other Effects

A second, less important, reason for us to be able match credit market data is the fact that our firms anticipate having to make costly equity issues in times of low profits to meet their financial obligations. As a result they will optimally choose somewhat lower leverage ratios ex ante. Since λ is only 2% in the baseline model this effect is relatively small here as we can see from Table 3.

As before we find that the persistence of the shocks retains a very important effect on prices. A low value for ρ_x variation in the market price of risk and thus credit spreads. This acts as an effective expansion in the supply of credit and which raises equilibrium leverage. Equilibrium default nevertheless falls because, with less persistent shocks, the option value of remaining an active firm rises. Our calibration of (rare) credit supply shocks is specifically designed to have only a negligible impact of these numbers, but it is easy to see how they can be used to generate more pronounced effects. Unsurprisingly larger variability in credit supply conditions will reduce average leverage but the overall effect on default rates and credit spreads depends on specific assumptions

[20] Assuming a realistic recovery rate of about 75% which our model is calibrated to match.

[21] Our decomposition is also consistent with Elton, Gruber, Agrawal and Mann (2001) who estimate that about two thirds of the credit spreads are due to the credit risk premium.

about the persistence of these shocks.

4.3 Investment and Finance over the Business Cycle

Table 4 documents the cyclical behavior of several investment and financing variables by reporting their cross-correlations with GDP. The table shows that all variables have the correct cyclical behavior in our baseline model although the implied correlations are sometimes higher than in the data. Intuitively this is because our benchmark calibration probably relies too much on a single source of aggregate uncertainty so that the innovations in output growth are too closely tied to those in aggregate productivity. Allowing for credit shocks in the last column generally produces more realistic numbers, with the possible exception of equity issues which become nearly uncorrelated with GDP growth. Intuitively movements in credit supply affect both leverage and spreads directly but GDP growth only through change in consumption and investment over time.

Persistence in aggregate shocks implies a more strongly pro-cyclical behavior in aggregate investment as new firms enter the market and build up productive capacity in anticipation of higher future profits. As a result the market value of firms (and especially of equity) is also more strongly pro-cyclical implying a countercyclical pattern in market leverage.

Also intuitive is the behavior of both default rates and credit spreads which are strongly countercyclical since default becomes less attractive when profits are temporarily high.

As in the data, our firms are more likely to issue equity during good times in the model, although the correlation with economic activity is relatively low. This is because firms must issue equity both at entry to finance investment in productive capacity and also when they need additional funds in times of low profits in order to cover coupon expenses. The former is usually stronger and so equity issues remain procyclical in our model. As the last two columns of Table 4 show however, lowering the issuance cost, λ, particularly in conjunction with large credit shocks can lead to countercyclical pattern in equity issuance.

4.4 Amplification and Asymmetry of Business Cycles

Figures **??** and **??** look at the impact of fluctuations in credit markets on key macroeconomic quantities. Figure **??** directly compares the response to exogenous technology shocks in our benchmark economy with levered firms, to the response in an alternative environment where all firms must be financed with equity alone. Unsurprisingly, we find that economic fluctuations are more pronounced in the levered economy, with both output, consumption and investment growth all responding between 35% to 50% more to an increase in the level of aggregate productivity. Again, we see that leverage introduces a powerful amplification mechanism since now positive productivity shocks reduce the risk of default and lower the cost of debt. This raises ex-ante firm value considerably and encourages firm creation and investment spending.[22] These amplifications results are quantitatively similar to those in Bernanke et al (1999). This is because both models are calibrated to deliver similar shares of investment in output and average credit spreads. However as we have seen, our transmission mechanism is different, and relies on movements in credit risk premium to produce the required variation in credit spreads and the cost of capital.

An important aspect of the model is the implied asymmetric in the response to shocks that is induced by the non-linear nature of entry and exit rules. Figure **??** illustrates these asymmetries in business cycle fluctuations with negative shocks producing sharper declines in output and investment. While a positive shock to productivity raises GDP growth about 1.2% above its mean, a negative shock of the same magnitude will reduce GDP growth by about 1.7%. Although often documented in the empirical literature this pronounced asymmetry in economic fluctuations is rarely obtained in general equilibrium macroeconomic models even when they do not rely on linear approximation to the stationary equilibria.[23]

Figure **??** helps explain this phenomenon. Intuitively a productivity shock has now two effects. First, a negative shock directly lowers output by lowering productivity. Second, it increases default rates and leads to a widening credit spreads. By making debt more expensive this further reduces the value of potential entrants and lowers investment spending. Both aspects are common to the

[22]Because we abstract from variations in labor supply these results are probably a lower bound on the amount of endogenous propagation that this mechanism can generate.

[23]Kyotaki and Moore (1997), Bernanke et al (1999), Cooley et al (2004).

financial accelerator literature but in our model the second effect is not symmetric because of role of risk premia in our model.

Figure **??** shows that risk premia acts in two ways. First as discussed earlier it makes the default rate respond asymmetrically to positive and negative shocks. Second, because bond losses are concentrated in recessions credit spreads widen more sharply here than in models with risk neutrality.[24] In turn this then exacerbates the response of aggregate investment to the negative productivity shock.

4.5 Credit Market Shocks

Although we have focused on the economy's response to technology driven fluctuations our framework can easily accommodate several other types of shocks. Figure **??** shows the predicted response of our economy to an unexpected tightening of credit conditions, i.e. an unexpected transition to the low ϕ state. This shock both devalues outstanding bonds, directly reducing household wealth, and makes new debt issues more costly, lowering investment and future output. The top panel confirms these predictions, showing that both output and investment fall, with output response actually peaking few quarters after the initial shock. The bottom panel also shows that default rates also rise, as the negative wealth effect feeds back to lower equity values of incumbent firms in general equilibrium.

Compared with the response to a standard technology shock a credit shock has a significantly larger impact on investment relative to output. Sensibly, this implies that in our model fluctuations in credit conditions impact young (and small) firms much more than the older established ones.

[24] Intuitively:

$$\text{Spread} = \text{Default Rate} \times \text{Loss in Default} \times \text{State Price of Default} \tag{33}$$

Because default losses are fixed by assumption, if risk premia does not move, movements in default rates and credit spreads are essentially one-to-one. Here however credit spreads move nearly twice as much as default rates.

4.6 Credit Spreads and Business Cycle Predictability

We now examine whether our model can match the well documented ability of credit spreads to forecast movements in the aggregate economy.[25] Table 5 shows the results of regressing the k period ahead growth in (log) output and investment, respectively, on the value weighted aggregate credit spread at time t for both our baseline model and in the data. Both panels show that credit spreads forecast movements in aggregate output and investment at horizons ranging between 1 quarter and 1 year. In both the data and the model the forecasts are usually statistically and economically meaningful. Moreover the estimated coefficients on the simulated panels are of very similar magnitudes to those found in recent empirical studies.

Table 6 shows that this predictability survives even after we control for the current state of aggregate productivity, x_t. Recall that in our model all aggregate variables such as output and investment depend on the two-dimensional aggregate state $s = (x, \mu(\cdot))$. The results in Table 6 show that, although important, variation in x_t is far from subsuming all of the information about the behavior of aggregate quantities. Unlike general aggregate models there is here an important role for firm heterogeneity and, to some extent, this is here captured by variation in credit spreads. This is not surprising since both $\mu(\cdot))$ and spreads contain much information about firm default probabilities and, as we have seen, these are closely tied to investment and output growth.

Table 7 recalculates the predictability regressions for the augmented version of our model with credit shocks. It shows how credit spreads become potentially more important in this world. Here they will reflect not only variation in cross-sectional default probabilities but also in exogenous credit conditions. Formally, with credit shocks the aggregate state space becomes $s = (x, \phi, \mu)$ and credit spreads now capture at least some of the variation in the last two. If, as Jermann and Quadrini (2010) argue, financial sector shocks are becoming more important, we would then expect the predictability of credit spreads to increase significantly.

[25] The forecasting ability of credit spreads is documented recently in Gilchrist et al(2008), Lettau and Ludvigson (2004) and Mueller (2008).

4.7 A Recession with Debt Overhang

The independent usefulness of credit spread information is also illustrated in Figures **??** and **??**. they compare the typical recession, depicted earlier in Figures **??** and **??**, with one that starts from a position of "debt overhang" where firms are exogenously endowed with excessive amounts of leverage.[26] Although somewhat arbitrary this experiment captures the possible aftermath of a "over-leverage" crisis like the one in 2008-09. Figure **??** shows how much deeper and more persistent this second recession would be in our model with GDP falling by an extra 0.5% per quarter and investment growth by almost 2%. Although both recessions are triggered by the same movement in aggregate productivity, their extent is only really captured by the sharp differences in credit spreads in Figure **??**.[27]

5 Conclusion

In this paper we propose a tractable general equilibrium asset pricing model with heterogeneous firms that endogenously links movements in stock and bond markets to macroeconomic activity. In our model these movements are associated to endogenous fluctuations in risk premia. As a result movements in financial variables such as credit spreads and expected equity returns will forecast future economic activity.

In our equilibrium setting, endogenous default increases the volatility of consumption during recessions, thereby rendering the market price of risk sharply countercyclical. As a consequence, expected returns on stocks and bonds are higher in recessions, raising the cost of capital and lowering investment and output growth. Endogenous movements in credit markets allow our model to match the observed conditional and unconditional movements in both financial prices and macroeconomic quantities in a parsimonious setting.

While a long theoretical literature in macroeconomics has demonstrated that financial frictions

[26] Formally we compare average recessions generated by a one standard deviation negative shock to x. In the first case all firms start with the optimal amount of leverage implied by their policy rules. In the second case we endow firms with the optimal amount of leverage implied by setting x one standard deviation above its mean.

[27] This figure also shows that in theory default rates - and indeed leverage - provide equally useful data that can be used to predict the response of the economy. In practice though default rates and leverage are measured with significant lags and imprecisions and tend not to be used as often as credit spreads by empiricists.

have the potential to deliver a powerful amplification mechanism for macroeconomic shocks, our focus on risk premia is quite distinct and, arguably, more appropriate to understand the recent developments in 2008-09.

References

Alvarez, Fernando and Urban Jermann, 2000, Efficiency, Equilibrium and Asset Pricing with Risk of Default, *Econometrica*, 775-797

Alvarez, Fernando and Urban Jermann, 2001, Quantitative Asset Pricing Implications of Endogenous Solvency Constraints, *Review of Financial Studies*, 1117-1152

Bansal, Ravi and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1639-1672

Bhamra, Harjoat, Lars-Alexander Kühn and Ilya Strebulaev, 2008, The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, forthcoming, *Review of Financial Studies*

Bhamra, Harjoat, Lars-Alexander Kühn and Ilya Strebulaev, 2008, The Aggregate Dynamics of Capital Structure and Macroeconomic Risk, forthcoming, *Review of Financial Studies*

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 1999, The Financial Accelerator in a Quantitative Business Cycle Framework, in *Handbook of Macroeconomics*, Edited by Michael Woodford and John Taylor, North Holland.

Brunnermeier, Markus and Sannikov, "A Macroeconomic Model with a Financial Sector", working paper, Princeton University, 2010.

Campanale, Claudio, Rui Castro and Gian Luca Clementi, 2008, Asset Pricing in a Production Economy with Chew-Dekel Preferences, working paper, New York University

Chang, Ganlin and Suresh Sundaresan, 2005, Asset Prices and Default-Free Term Structure in an Equilibrium Model of Default, *Journal of Business*, 78, 1215-1266

Chen, Long, Pierre Collin-Dufresne and Robert Goldstein, 2008, On the Relation between Credit Spread Puzzles and the Equity Premium Puzzle, forthcoming *Review of Financial Studies*

Chen, Hui, 2008, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, forthcoming, *Journal of Finance*

Cooley, Thomas, Ramon Marimon and Vincenzo Quadrini, 2004, Aggregate Consequences of Aggregate Contract Enforceability, *Journal of Political Economy*, 112, 817-847

Covas, Francisco and Wouter Den Haan, 2007, The Role of Debt and Equity over the Business Cycle, working paper, University of Amsterdam

Croce, Massimiliano, 2010, Long-Run Productivity Risk: A New Hope For Production-Based Asset Pricing?, working paper, University of North Carolina, Chapel Hill

Fama, Eugene F., and Kenneth R. French, 1992, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics*, 25, 23-49.

Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics*, 33, 3–56.

Gala, Vito, 2007, Investment and Returns, working paper, London Business School

Gilchrist, Simon, Vladimir Yankov and Egon Zakrajsek, 2008, Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets, working paper, Boston University

Gomes, João F., 2001, Financing Investment, *American Economic Review*, 90, 5, 1263-1285.

Gomes, João F., Leonid Kogan, and Motohiro Yogo, 2008, Durability of Output and Expected Stock Returns, forthcoming, *Journal of Political Economy*

Gomes, João F., Leonid Kogan, and Lu Zhang, 2003, Equilibrium Cross-Section of Returns, *Journal of Political Economy*, 111, 693-731.

Gomes, João F., Amir Yaron, and Lu Zhang, 2006, Asset Pricing Implications of Firm's Financing Constraints, *Review of Financial Studies*, 19, 1321-1356.

Gomes, João F., and Lukas Schmid, 2008, Levered Returns, forthcoming *Journal of Finance*

Gourio, Francois, 2010, Credit Risk and Disaster Risk, working paper, Boston University.

Hackbarth, Dirk, Jianjun Miao and Erwan Morellec, 2006, Capital Structure, Credit Risk and Macroeconomic Conditions, *Journal of Financial Economics*, 82, 519-550

Hennessy, Christopher, and Amnon Levy, 2007, Why does Capital Structure Choice Vary with Macroeconomic Conditions? , *Journal of Monetary Economics*, 54, 1545-1564

Hennessy, Christopher, and Toni Whited, 2005, Debt Dynamics *Journal of Finance*, 60, 1129-1165

Jermann, Urban, 1998, Asset Pricing in Production Economies, *Journal of Monetary Economics*

Jermann, Urban and Vincenzo Quadrini, 2010, Macroeconomic Effects of Financial Shocks, working paper, University of Pennsylvania.

Jermann, Urban and Vivien Yue, 2007, Interest Rate Swaps and Corporate Default, working paper, University of Pennsylvania.

Kaltenbrunner, Georg and Kars Lochstoer, 2008, Long Run Risk through Consumption Smoothing, forthcoming, *Review of Financial Studies*

Kiyotaki, Nobu and John Moore, 1997, Credit Cycles, *Journal of Political Economy*, 105, 211-248

Keim, Donald and Robert Stambaugh, 1986, Predicting Returns in the Stock and Bond Markets, *Journal of Financial Economics*, 17, 357-390

Khan, Aubhik, and Julia Thomas, 2003, Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter? *Journal of Monetary Economics*, 50, 331-360

Krusell, Per and Anthony Smith, 1998, Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, 106, 867-896

Kühn, Lars-Alexander, 2008, Asset Pricing Implications of Real Investment Commitment, working paper, Carnegie Mellon University

Leland, Hayne, 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance*, 49, 1213-1252

Lettau, Martin and Sydney Ludvigson, 2002, Time-Varying Risk Premia and the Cost of Capital: An Alternative Implication of the Q-Theory of Investment, *Journal of Monetary Economics*, 49, 31-66

Miao, Jianjun and Pengfei Wang, Credit Risk and Business Cycles, working paper, Boston University, 2010.

Müller, Philippe, 2008, Credit Spreads and Real Activity, working paper, London School of Economics

Philippon, Thomas, 2008, The Bond Market's Q, forthcoming, *Quarterly Journal of Economics*

Schwert, William, 1989, Why does Stock Market Volatility Change over Time?, *Journal of Finance*, 44, 1115-1153

Stock, James and Mark Watson, 1999, Business Cycle Fluctuations in U.S. Macroeconomic Time Series, *Handbook of Macroeconomics*

Strebulaev, Ilya, 2007, Do Tests of Capital Structure mean what they say?,*Journal of Finance*, 62, 1747-1788

Sundaresan, Suresh, and Neng Wang, 2006, Dynamic Investment, Capital Structure, and Debt Overhang, working paper, Columbia University

Tallarini, Thomas, 2000, Risk-sensitive Real Business Cycles, *Journal of Monetary Economics*

Appendix: Computation Details

Computation of the competitive equilibrium is complicated by the endogeneity of the pricing kernel, which embodies the equilibrium market clearing conditions. The main difficulty here is the dependence of all aggregate quantities on the cross-sectional distribution, μ, a high-dimensional object.

Our solution algorithm exploits the parsimonious characterization of the distribution μ and relies on two basic techniques. First, we re-normalize the value functions for debt and equity to express them in units of marginal utility which is computationally more convenient. Next, following Krusell and Smith (1998), the cross-sectional distribution μ is approximated by a low-dimensional state variable that summarizes the relevant information in μ.

The expression for the pricing kernel (25) guides both our choice of the approximate state space and the re-normalizations. To that end, we define the function:

$$p(C, W) = C^{-\frac{\kappa}{\sigma}} W^{\kappa-1}$$

and rewrite the expression for the market-to-book value of equity as

$$\hat{Q}(s, z, b) \quad = \quad Q(s, z, b)p(C, W)$$

The normalized market value of debt (relative to capital), $\hat{B}(s, z, b)$, is defined in the same way. Our numerical strategy is based on numerically iterating on these two to obtain individual policy functions and then aggregate. Since both values depend on the aggregate state $s = (x, \mu)$ we start by approximating its high-dimensional space by $\hat{s} \equiv (x, W)$. In other words, we assume that aggregate household wealth W captures the relevant information about aggregate quantities contained in the cross-sectional distribution μ. We can then write the equity value of the firm as:

$$\begin{aligned}\hat{Q}(x, W, z, b) \quad = \quad & \max\{0, (1-\tau)(1-\lambda)(xz - b)p(\hat{C}(x, W), W) \\ & + E\left[\beta^{\kappa}\left(\frac{W' + \hat{C}(x', W')}{\hat{C}(x, W)}\right)^{\kappa-1} \hat{Q}(x', W', z', b)\right]\}\end{aligned}$$

where $\hat{C}(x, W) = C(\hat{s})$ in our approximate state space. $\hat{B}(x, W, z, b)$ is defined analogously.

Again following Krusell and Smith (1998) we parameterize both the consumption policy $\hat{C}$ and the law of motion for aggregate wealth W' as log linear functions of the aggregate state, x and W:

$$\begin{aligned} \log C &= \alpha_0 + \alpha_1 \log x + \alpha_2 \log W \\ \log W' &= \eta_0 + \eta_1 \log x + \eta_2 \log W \end{aligned}$$

for some coefficient vectors α and η. With these rules at hand we compute firm value and policies. These are then aggregated and checked for consistency using the general equilibrium condition (28).

More precisely, we use the following iterating procedure:

- Discretize the state space by choosing discrete grids for b and W, and the shocks x and z.[28],[29]
- Guess initial vectors α^0 and η^0
- Iterate on the functional equations for $\hat{Q}$ and $\hat{B}$ and compute decision rules for investment, default and leverage.
- Simulate decisions rules and compute the implied equilibrium allocations for C and W.
- Use implied time series for x, C and W to revise log linear rules for C and W' and check fit.
- Iterate until convergence.

The simulation uses the law of motion for the cross-section of firms over time defined by equation (29). The key step here is to recognize that at any point in time μ_t can be represented by a matrix of size $nz \times nb$, where each element $\mu_{t,ij}$ represents the mass of firms with coupon b_i which drew idiosyncratic shock z_j at time t. The default decision is efficiently characterized by the selection of μ_t implied by the optimal policy χ, while entry adds a vector corresponding to the ergodic distribution of $G(z)$ at the position implied by the optimal choice of leverage for new entrants at time t and mass equal to $h.F(s)$.

[28]As b is a function of both x and wealth W, so $nb = nx \times nw$, where n_i is the number of points in the grid for $i = b, z, x, W$.

[29]We use the procedure in Rouwenhorst (1995) since are highly persistent.

Table 1: **Calibration**

Parameter Values	
Preferences	
β	0.993
γ	10
σ	2
Technology	
δ	0.05
ρ_x	0.96
σ_x	0.015
ρ_z	0.9
σ_z	0.18
Institutions	
τ	0.2
λ	0.02
ϕ	0.4

This table reports the basic parameter choices for our model. These choices are discussed in detail in Section 3.2. The model is calibrated at quarterly frequency to match data both at the macro level and in the cross-section.

Table 2: **Aggregate Moments**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Macro Moments							
$\sigma[\Delta_C]$	1.68	1.64	1.27	1.53	1.79	1.61	1.74
$\frac{\sigma[\Delta_C]}{\sigma[\Delta_Y]}$	0.70	0.61	0.63	0.68	0.75	0.60	0.66
$\frac{\sigma[\Delta_I]}{\sigma[\Delta_Y]}$	4.59	3.73	3.85	3.48	2.91	3.74	3.82
$\frac{I}{Y}$	0.19	0.23	0.20	0.20	0.18	0.24	0.23
Asset Pricing Moments							
$E[r^f]$	1.69	1.34	2.11	3.64	1.90	1.38	1.19
$\sigma[r^f]$	2.21	1.59	1.04	0.92	1.14	1.55	1.46
$E[r^e - r^f]$	4.29	4.20	1.43	1.35	2.93	4.24	4.03
$\sigma[r^e]$	17.79	10.86	3.76	3.97	6.62	10.97	11.12

This table reports unconditional sample moments generated from the simulated data of some key variables of our model under different parameter specifications. We report averages across 1000 simulations of 59 years. All data are annualized. The return on equity refers to the value weighted aggregate stock market return. The parameter values used in the benchmark simulation are reported in table 2. Data counterparts come from the BEA and CRSP

Table 3: **Credit Market Statistics**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Default Rate	1.48%	1.42%	0.00	0.60%	0.97%	1.29%	1.58%
Credit Spread	0.95%	1.04%	0.00	0.43%	0.80%	0.91%	1.12%
Market Leverage	0.35	0.33	0.00	0.44	0.41	0.38	0.31

This table reports statistics related to credit markets and firms' capital structures. Data from the model comes from averages across 1000 simulations of 59 years. All data are annualized. The default rate is from Jermann and Yue (2007). The average (annualized) credit spread is the AAA-BAA spread. This data and that for the market leverage ratios are for the period between 1951 and 2008 and come from the Board of Governors of the Federal Reserve.

Table 4: **Financing Over Business Cycle**

Variable	Data	Benchmark	All Equity	$\rho_x = 0.9$	$h'(x) = 0$	$\lambda = 0$	Credit Shocks
Investment	0.81	0.72	0.77	0.66	0.69	0.74	0.52
Market leverage	-0.11	-0.60	0.00	-0.51	-0.56	-0.61	-0.33
Equity Issuance	0.10	0.19	0.28	0.14	0.09	0.17	0.03
Default rate	-0.33	-0.81	0.00	-0.84	-0.75	-0.80	-0.59
Credit Spread	-0.36	-0.74	0.00	-0.78	-0.82	-0.72	-0.42

This table reports business cycle properties of key macro and financial variables in the model. Data from the model comes from averages across 1000 simulations of 59 years. For flow variables we use correlations between growth rates. For leverage we report correlation with end of period ratios. Empirical sources are the Bureau of Economic Analysis and the Board of Governors of the Federal Reserve.

Table 5: **Forecasting Output and Investment Growth**

	Data		
Horizon k	1 quarter	2 quarter	1 year
$\Delta Y_{t,t+k}$	-1.20	-2.24	-3.89
	(-4.26)	(-3.58)	(-2.82)
$\Delta I_{t,t+k}$	-3.69	-6.54	-9.71
	(-4.30)	(-3.16)	(-2.39)
	Model		
Horizon k	1 quarter	2 quarter	1 year
$\Delta Y_{t,t+k}$	-1.69	-2.19	-2.85
	(-2.41)	(-2.28)	(-2.15)
$\Delta I_{t,t+k}$	-2.92	-5.73	-7.96
	(-2.32)	(-2.21)	(-2.07)

This table reports forecasting regressions for output and investment growth in both the model and the data. We regress k-period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+k} = \log I_{t+k} - \log I_t$ on the value weighted aggregate credit spread at time t. T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.

Table 6: **Forecasting with Credit Spreads - Baseline Model**

	CS_t	x_t	R^2
$\Delta Y_{t,t+1}$	-1.69		0.22
	(-2.41)		
		1.44	0.65
		(2.92)	
	-0.83	2.06	0.79
	(-2.16)	(2.54)	
$\Delta I_{t,t+1}$	-2.92		0.11
	(-2.32)		
		4.58	0.49
		(3.27)	
	-1.55	6.19	0.62
	(-2.03)	(2.40)	

This table reports forecasting regressions for output and investment growth in the baseline model. We regress 1 period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+1} = \log I_{t+1} - \log I_t$, on the value weighted aggregate credit spread at time t, CS_t, and the aggregate shock at time t, x_t. T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.

Table 7: **Forecasting with Credit Spreads - Credit Shocks**

	CS_t	x_t	R^2
$\Delta Y_{t,t+1}$	-1.25		0.26
	(-2.38)		
		1.56	0.55
		(2.29)	
	-1.14	1.91	0.74
	(-2.47)	(2.43)	
$\Delta I_{t,t+1}$	-3.11		0.23
	(-2.57)		
		3.82	0.48
		(2.27)	
	-1.78	4.46	0.66
	(-2.12)	(2.09)	

This table reports forecasting regressions for output and investment growth in the model with credit shocks. We regress 1 period ahead log growth in output and investment, respectively: $\Delta Y_{t,t+k} = \log Y_{t+k} - \log Y_t$ and $\Delta I_{t,t+1} = \log I_{t+1} - \log I_t$, on the value weighted aggregate credit spread at time t, CS_t, and the aggregate shock at time t, x_t. T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 59 years. Standard errors are corrected using Newey-West with 4 lags.