Feedback Trading between Fundamental and Nonfundamental Information

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We develop a strategic trading model in which an insider exploits noise traders' overreaction. A feedback effect arises from the insider's trading on fundamental information (the expected growth rate of dividends) and nonfundamental information (insider's inventory or noise supply). We find that the stock price is not fully revealing; a faster mean-reverting noise supply leads to a more volatile price; the price impact can increase with insider's risk-aversion; and a risk-averse insider can trade more aggressively on fundamental information than a risk-neutral one does. Insider's current trade and his previous inventory exhibit simultaneously positive forecasting powers for future stock returns. (*JEL* D82, D84, G11, G12, G14)

In this paper, we present an intertemporal asset pricing model in which a monopolistic risk-averse insider exploits mean-reverting noise supply. Our study is motivated by De Bondt and Thaler's (1985, 1987) hypothesis of investor overreaction. These two papers find systematic price reversals for stocks experiencing extreme gains or losses over the long term: past losers significantly outperform past winners. They interpret these results to be consistent with the hypothesis of investor overreaction; that is, individual investors or noise traders tend to overreact to unexpected new information; as new information arrives, the initial bias of noise traders due to excessive optimism or pessimism gets corrected through their trading.[1] This overreaction hypothesis suggests that the aggregate position of noise traders (noise supply) tends to revert. In other words, noise traders' overreaction implies a mean-reverting noise supply.[2]

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[1] Investor overreaction is supported by the experimental psychological research of Kahneman and Tversky (1982).

[2] We thank the referee for this suggestion. The mean-reverting speed of the noise supply represents the correction speed of noise traders' overreaction.

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There is a large body of empirical literature following De Bondt and Thaler (1985, 1987) on price reversals; however, there are few theoretical studies on the effects of investor overreaction. In Campbell and Kyle's (CK; 1993) and Wang's (1993) models, rational risk-averse investors absorb noise shocks and thus provide liquidity to noise traders whose aggregate position is mean reverting.[3] As a result, rational investors generally do not exploit noise traders' overreaction behavior and stabilize the stock price in these models, though they can infer the noise supply in equilibrium. These results are inconsistent with the empirical evidence of Brunnermeier and Nagel (2004) and of Froot and Ramadorai (2005), in which sophisticated (rational) investors, such as hedge funds, take advantage of the predictable patterns of noise trading and thus destabilize the stock price.

We take a new approach to examine the effect of noise traders' overreaction, wherein rational investors exploit the mean-reverting noise supply and can destabilize the stock price. To do so, we extend the strategic trading framework of Kyle (1985) by introducing a monopolistic insider who enjoys information advantages over market makers regarding the expected growth rate of dividends and the mean-reverting noise supply in an infinite-horizon continuous-time model.[4] More generally, the insider trades on his fundamental and nonfundamental information advantages. Here, fundamental information refers to information related directly to the fundamentals of an asset, including the expected growth rate of dividends and the dividend itself, and nonfundamental information refers to information that is not related directly to fundamentals, including the noise supply and informed traders' own inventory position.

Importantly, there exists a self-reinforcing mechanism between insider's trading on his fundamental information and that on his nonfundamental information in our model, which we term the *feedback effect*. Insider's trading on nonfundamental information makes it more difficult for market makers to infer fundamental information, allowing more aggressive trading by the insider on fundamental information than in the case in which the insider has no advantage on nonfundamental information.[5] We shall demonstrate that this feedback effect leads to several novel results.

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[3] CK establish a link between noise traders' overreaction (proxied by a mean-reverting noise supply) and price reversals, consistent with the empirical findings of De Bondt and Thaler.

[4] Kyle (1985) analyzes an insider's optimal trading in a finite-horizon nonstationary setting, in which the insider receives only a single piece of private signal regarding an asset's payoff at the beginning of trading. Kyle demonstrates the trade-off between trading aggressiveness and price impact. Much of the subsequent literature follows this assumption. See, for example, Back (1992), Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), Spiegel and Subrahmanyam (1992), Foster and Viswanathan (1996), Back and Pederson (1998), Back, Cao, and Willard (2000), Baruch (2002), Bernhardt and Miao (2004), and Seiler and Taub (2008). For comprehensive reviews, see O'Hara (1995), Biais, Glosten, and Spatt (2005), and Vives (2010).

[5] Note that insider's trading on fundamental information also makes it more difficult for market makers to infer nonfundamental information, allowing more aggressive trading by the insider on nonfundamental information.

Specifically, we consider an infinite-horizon continuous-time model with a risk-averse insider, competitive risk-neutral market makers, and a mean-reverting noise supply, in which new information regarding dividends and their expected growth rate arrives at the market continuously. The insider has monopolistic power in observing the expected growth rate of dividends. Following Kyle (1985), Back (1992), Vayanos (2001), and Chau and Vayanos (CV, 2008), we adopt a smooth trading strategy for the insider because he trades strategically by taking into account the price impact. To ensure a bounded solution, we assume that the insider incurs a trading cost that is a quadratic function of the quantity that he trades. For comparison with the previous literature, we focus on the limiting case in which the quadratic cost coefficient goes to zero.

Two recent papers also study the trading by a monopolistic insider in infinite-horizon steady-state models, but they do not consider overreacting noise traders or a mean-reverting noise supply. Vayanos (2001) considers a risk-averse insider who possesses private information regarding his own time-varying endowment and trades with risk-averse market makers. In CV (2008), private information on the expected growth rate of a dividend process arrives in the market repeatedly, all agents are risk neutral, and the noise supply follows a random walk. Strikingly, both papers show that the insider who trades infinitely aggressively is impatient and the private information tends to be fully revealing in the continuous-time limit.[6] In particular, Vayanos (2001) shows that risk aversion does not prevent the insider from trading infinitely aggressively on his private information, and consequently, the equilibrium price still becomes fully revealing.

As in Vayanos (2001) and CV (2008), the insider is impatient in our model. It is then unclear whether the insider would instantaneously reveal his information advantages even in the presence of a mean-reverting noise supply. Our model is able to study this issue rigorously. Similar to Kyle (1985), Back (1992), and CV (2008), the insider in our model trades on his fundamental informational advantage. In addition, because the insider is risk averse, he hedges the risk of his future investment opportunities by trading against his own inventory in the stock, as well as by reducing his trading on fundamental information (hedging effect). Specifically, the risk-averse insider reduces (increases) his inventory when insider's inventory is high (low), and as a result, insider's inventory is mean reverting. Because the insider knows his own inventory, he can infer the noise supply perfectly from the price. Because of their mean-reverting features, the noise supply and insider's inventory are both predictable from their historical values. The insider can then estimate these variables and

[6] Intuitively, a steady-state equilibrium ensures a constant price impact. The insider is impatient to take advantage of his private information, because old information loses its value with perishable new information arriving. Given a constant price impact, he tends to trade infinitely aggressively on market makers' estimation error of the private information in the continuous-time limit, leading to a fully revealing equilibrium.

the total order flow in future periods more accurately than can market makers. Hence, the insider has information advantage over market makers regarding the future realizations of insider's inventory or the noise supply, and equivalently, the insider has information advantage regarding the future realizations of the total order flow.

Because the insider in our model trades strategically and smoothes his order over time, market makers observe a variable (equivalent to the accumulated order flow), which is a function of the noise supply and the expected growth rate of dividends in both the current period and the previous periods.[7] As we explain in Section 2.3, this historical dependency enables the insider to trade on both his fundamental and his nonfundamental information advantages over market makers, and consequently, the insider exploits his information advantage regarding noise traders' overreaction.[8]

Because of the feedback effect, the insider camouflages his fundamental information by trading on his nonfundamental information. As the transaction cost goes to zero, the insider's trades on fundamental information and nonfundamental information approach infinity in the same order. As a result, market makers cannot distinguish between the two types of information. Consequently, the equilibrium stock price is not fully revealing. This result holds even when the insider is risk neutral but noise traders overreact to new information (the noise supply is mean reverting), in which there is only a feedback effect. This nonfully revealing result is consistent with the empirical evidence of Meulbroek (1992), but in contrast to that of Vayanos (2001) and CV (2008), in which equilibrium prices are fully revealing in the continuous-time limit.[9] In addition, the feedback effect contributes to the "excess price volatility" of a stock compared with the simple present value model with a constant discount rate (see Campbell and Shiller, 1988; Fama and French, 1988).[10]

When the correction speed of noise traders' overreaction increases (an increase in the mean-reverting speed of the noise supply), the amount of noise trading (measured by the variance of the quantity traded by noise traders) in any

[7] Recall that in Wang's (1993) work, uninformed investors observe the price or equivalently a linear function of the noise supply and the expected growth rate of dividends in the current period. Because informed traders' information advantages regarding the noise supply and the expected growth rate of dividends are perfectly negatively correlated, informed investors trade only on their information advantage on the fundamentals, and trading on their information advantage regarding the noise supply does not add value to them.

[8] As market makers observe the total order flow, their estimation errors regarding insider's inventory and the noise supply are perfectly negatively correlated. Equivalently, insider's trading on nonfundamental information is either on market makers' estimation error regarding the noise supply or on that regarding insider's inventory.

[9] The empirical evidence of Meulbroek (1992) suggests that the market infers informed trading and impounds a large portion of it, but the market does not incorporate all of the private information into the stock price before the private information becomes public.

[10] To see this point, consider risk-neutral investors in both competitive and strategic models, such as those studied by CK, Wang, and CV. Because the equilibrium price is fully revealing, the price volatility is equal to the volatility of the fundamental value. Because of the feedback effect, the equilibrium price is more volatile in our model than in these models when the insider is risk neutral.

interval decreases. Because of the feedback effect, however, the insider trades more aggressively on both nonfundamental and fundamental information, leading to a less informative and more volatile price. Economically, we demonstrate that rational traders can destabilize prices in the presence of overreacting noise traders. In contrast, in the work of CK (1993) and Wang (1993), noise trading does contribute to the excess price volatility, but when the correction speed of noise traders' overreaction increases, the price becomes less volatile. Intuitively, when the noise supply reverts to its long-term value faster, the amount of noise risk declines. Because informed traders provide liquidity to noise traders, they would require a smaller risk premium, leading to a smaller price variability. Informed traders thus stabilize the price in these models.

Because of the feedback effect, our model also generates interesting results on the relationship between insider's risk aversion and the price impact. The traditional view is demonstrated by Subrahmanyam (1991) and Baruch (2002); that is, the price impact always decreases with insider's risk aversion, because a more risk-averse insider trades less aggressively on his fundamental information because of the hedging effect. In our model, their result holds only when the amount of fundamental information is small or the insider is very risk averse (the hedging effect dominates). When the amount of fundamental information is large, however, a rise in the risk aversion of the insider can lead to a larger price impact (the feedback effect dominates). Strikingly, a risk-averse insider may trade more aggressively on his private fundamental information than a risk-neutral one does in equilibrium.

The feedback effect in our model is due to either the mean reversion of the noise supply or the risk aversion of the insider. There are similarities and differences in the effects of these two mechanisms on the properties of the equilibrium. Because of the feedback effect, an increase in the mean-reverting speed of the noise supply or the risk aversion of the insider causes the insider to trade more aggressively on nonfundamental information, leading to a less informative and more volatile price, as well as a smaller (larger) price impact when the amount of private information is small (large). The main difference is that the mean reversion of the noise supply generates a pure feedback effect, whereas the insider's risk aversion induces both a hedging effect and a feedback effect. As a result, for the risk-aversion mechanism, the value of private information is less than that when the insider is risk neutral and the noise supply follows a random walk; the expected insider's trade based on market makers' information is nonzero; and the insider can trade positively on market makers' estimation errors regarding the noise supply.

Our model generates unique empirical implications. First, our model predicts that a faster correction speed of noise traders' overreaction leads to a higher idiosyncratic volatility of a stock.[11] Second, our model predicts that a faster

[11] Because private information is more likely to be firm specific, we should use the idiosyncratic volatility in empirical tests and control for the price volatility due to systematic factors and industry factors.

correction speed of noise traders' overreaction leads to more aggressive trading by the insider on his information advantages regarding both fundamental information and noise supply. The second prediction is about the existence of the feedback effect. These predictions can be tested using data on the holdings of institutional investors (proxy for the insider) and individual investors (proxy for noise traders). In addition, because the insider is risk averse in our model, he trades against his inventory. As a result, insider's order in the current period and his inventory in the previous period are both positively related to future stock returns, consistent with the empirical findings of Bennett, Sias, and Starks (2003), Yan and Zhang (2009), and Baik, Kang, and Kim (2010). These results cannot be obtained by previous strategic trading models, such as Kyle (1985), Back (1992), Vayanos (2001), and CV (2008). Our model further predicts that the forecasting power on future stock returns by insider's trade in the current period increases with the correction speed of noise traders' overreaction, whereas the forecasting power on future stock returns by insider's holdings in the previous period decreases with it.

Our paper is related to the literature on the high stock price volatility. CK (1993) and Wang (1993) rely on exogenous noise supply; Campbell and Cochrane (1999) resort to a habit-formation argument; and Veronesi (1999) employs information uncertainty to explain the high price volatility. We demonstrate that a feedback effect arising from insider's trading on fundamental and nonfundamental information contributes to additional price volatility.

Our paper is also related to the literature on multidimensional information under both strategic and competitive paradigms. Rochet and Vila (1994) investigate a static Kyle-type model in which the insider knows the noise beyond his fundamental information. Because this is a static model, there is no feedback effect. Ganguli and Yang (2009) and Manzano and Vives (2011) extend Grossman and Stiglitz's (1980) work by allowing the informed traders to observe the noise supply. Because the price is a linear function of the fundamental information and the noise supply, the estimation errors on the fundamental information and the noise supply are perfectly correlated. As a result, there is no feedback effect. Amador and Weill (2010) consider an economy in which workers observe a private signal regarding productivity and learn monetary shocks from public prices. They show that the feedback effect between aggregate price and labor supply may lead to a reduction in the price informativeness. Goldstein, Ozdenoren, and Yuan (2011) study the informational feedback from the trading of currency speculators to policy decisions of central bankers. They show that the learning process from the aggregate trading of speculators can give rise to coordination motives among speculators, leading to large currency attacks and introducing nonfundamental volatility into exchange rates. These papers do not study the feedback effect between the trading on fundamental information and that on nonfundamental information in an infinite-horizon model.

1. Model Setup

We consider a continuous-time model over an infinite horizon $(-\infty, \infty)$. There is one consumption good and two financial assets: a risk-free bond and a risky stock. Three types of investors exist in this economy: an insider, noise traders, and market makers.

1.1 Assets and fundamental information

The bond yields an exogenous and constant rate of return r $(r > 0)$. Each share of stock generates a flow of dividends at an instantaneous rate D_t at time t. Adopting from CK (1993), we assume that D_t follows a Brownian motion with a time-varying mean, according to the processes

$$dD_t = \alpha_I I_t dt + \sigma_D dB_{1t}, \tag{1}$$

$$dI_t = -\alpha_I I_t dt - \eta\sigma_D dB_{1t} + \sqrt{2\eta - \eta^2}\sigma_D dB_{2t}. \tag{2}$$

Here, α_I is a positive constant, and B_{1t} follows a standard Brownian motion process. The process I_t reverts to zero. η and σ_D are positive constants, with $0 \le \eta < 2$; B_{2t} is a standard Brownian motion process independent of B_{1t}; and the parameter α_I measures the reverting rate. $\alpha_I I_t$ can be interpreted as the expected growth rate of dividends, which is observed only by the insider, whereas D_t is public information known to all investors in the economy. The information processes in Equations (1) and (2) ensure the following equations:

$$E\{I_{t+s} | D(-\infty, t]\} = 0, \quad E\{D_{t+s} | I(-\infty, t]\} > 0, \qquad s \ge 0. \tag{3}$$

Equation (3) means that the history of the D_t process cannot forecast the future values of I_t but the history of the I_t process can forecast the future values of D_t.[12]

Following CK (1993), we define the fundamental value V_t as the discounted value of expected future dividends, conditional on all available information in the economy up to time t:

$$V_t = E\left[\int_0^\infty \exp(-rs) D_{t+s} ds | \mathcal{F}(t)\right], \tag{4}$$

where $\mathcal{F}(t)$ denotes the σ-field generated by $\{(D_s, I_s) : s \in (-\infty, t]\}$.[13] In the Appendix, we show that V_t is a linear function of D_t and I_t. We thus term

[12] The assumption that the mean-reverting speed of I_t and the coefficient of I_t in the drift of D_t are equal may seem restrictive. However, as shown by Campbell and Kyle (1993), given Equation (3) and the fact that D_t and I_t follow a continuous-time VAR process, the two-dimensional processes of D_t and I_t are uniquely determined after rescaling. Wang (1993) considers an information structure given by $dD = Idt + b_1 dB_1$ and $dI = -\alpha_I I dt + b_2 dB_2$, where B_1 and B_2 are two independent standard Brownian motions. We can prove that after a reacaling exercise, the processes in Wang are equivalent to Equations (1) and (2).

[13] In CK, because of the CARA utility function, the risk aversion of investors increases the expected return on a stock by reducing the stock price via a separate risk premium term rather than an increase in the discount rate.

D_t and I_t *fundamental information*. In particular, as I_t is known only to the insider, it is termed *private fundamental information*.[14] Because the ratio of the instantaneous variance of I_t to that of D_t equals 2η, we interpret parameter η as a measure of the relative amount of private fundamental information.

1.2 Noise traders, the insider, and nonfundamental information

Following CK (1993) and Wang (1993), we assume that the supply of noise traders (hereafter noise supply) follows a mean-reverting process:

$$dU_t = -aU_t dt + \sigma_U dB_{3t}, \tag{5}$$

where a and σ_U are positive constants and B_{3t} is a standard Brownian motion process independent of B_{1t} and B_{2t}. A mean-reverting noise supply is commonly adopted in the literature. As discussed already, we interpret a mean-reverting noise supply as the overreaction of noise traders to new information, in which parameter a measures the mean-reverting or the correction speed of noise traders' overreaction.

The insider has monopoly power over I_t. He has an exponential utility of the form:

$$U(C_t) = -\frac{1}{\gamma}\exp(-\rho t - \gamma C_t), \tag{6}$$

where ρ is the time-preference parameter; γ is the risk aversion coefficient; and C_t is the consumption rate at time t. The insider chooses C_t and a trading strategy to maximize his expected utility over an infinite time horizon, conditional on his information set.

Kyle (1985), Back (1992), Vayanos (2001), and CV (2008) demonstrate that the insider adopts a smooth trading strategy in the continuous-time limit because he trades strategically by taking into account the price impact. Following these papers, we consider the form of insider's trading strategy to be

$$dX_t = \theta_t dt, \tag{7}$$

where X_t denotes his stock inventory at time t.[15] Hence, the insider chooses the order rate θ_t, whereas X_t is a state variable. The insider does not observe noise traders' inventory U_t directly. Because he observes the history of the dividend, the private fundamental information, his own inventory, and the stock price up to time t, he can infer U_t perfectly from the price. Hence, the information set of the insider at time t is $\mathcal{F}_I(t)$, the σ-field generated by $\{(D_s, I_s, U_s) : s \in (-\infty, t]\}$.

It is intractable for us to solve the model in discrete time when the insider is risk averse. For tractability, we solve the model in continuous time. To

[14] We simply refer to I_t as fundamental information whenever it does not cause confusion in the rest of the paper.

[15] The intuition for this smooth-trading strategy is as follows. Suppose the price before trading is P_-. After the insider submits an order of dX, the price is given by $P_- + dP$. The market impact cost on the trade is then given by $dPdX$, which is in the order of $(dt)^{3/2}$. If there were a diffusion part in dX, then $dPdX$ would be in the order of dt, increasing the market impact cost significantly.

ensure a bounded solution, we assume that the insider incurs a quadratic trading cost whose rate is given by $\frac{1}{2}k\theta_t^2 dt$ at time t, where k is a positive constant. Subrahmanyam (1998) has considered this type of cost, and interprets it as a tax associated with insider's trade size. In essence, the adoption of this cost function is a modeling device allowing us to obtain tractable interior solutions.

As X and U contain information that is not related directly to the fundamental value V, we term them *nonfundamental information*.

1.3 Market makers and the stock price function

Market makers are risk neutral. At each time t, the insider and noise traders submit market orders to market makers, who observe the accumulated order flows ω and dividend information D up to time t, where $\{\omega_s \equiv X_s + U_s, s \in (-\infty, t]\}$. They set the stock price competitively, at which they trade with the insider and noise traders, conditional on their information set $\mathcal{F}_M(t)$, the σ-field generated by $\{D_s, \omega_s : s \in (-\infty, t]\}$ at time t. They earn zero expected profits. In the Appendix, we show that the equilibrium stock price is given by

$$P_t = E\left[\int_{s=0}^{\infty} \exp(-rs) D_{t+s} ds | \mathcal{F}_M(t)\right] = \frac{D_t}{r} + \mu \widehat{I}_t, \tag{8}$$

where $\mu = \frac{\alpha_I}{r(r+\alpha_I)}$ and $\widehat{I}_t \equiv E[I_t | \mathcal{F}_M(t)]$.

The first component in the price function represents the discounted value of expected future dividends based on dividend information at time t. The second one represents the discounted value of expected future private fundamental information based on market makers' information set. As I_t follows a mean-reverting process, the parameter μ depends on both r and α_I. Risk-neutral market makers allow us to focus on the impact of the feedback effect on the price dynamics under information asymmetry. A model in which market makers are risk averse is considered by Guo and Kyle (2012).

2. Equilibrium

This section solves for the equilibrium of the economy. An equilibrium consists of an insider's trading strategy $\{\theta_t\}_{t\in(-\infty,\infty)}$, his optimal consumption rate $\{C_t\}_{t\in(-\infty,\infty)}$, and a price function $\{P_t\}_{t\in(-\infty,\infty)}$. Given the pricing rule and market makers' updated beliefs, the insider chooses his consumption and order rates optimally by taking into account the impact of his trades on the equilibrium price. Given insider's trading strategy, market makers then solve a Kalman filtering problem to update their expectations about the underlying state variables, setting the price efficiently. We consider only a linear equilibrium, in which insider's order rate is a linear function of his state variables. Similar to Vayanos (2001) and CV (2008), we consider a steady-state equilibrium in which these functions are time independent.

2.1 Insider's candidate strategy and market makers' filtering problem

Market makers learn about the values of private fundamental information I_t, noise supply U_t, and insider's inventory X_t, given the history of dividends and order flows to time t. In equilibrium, the order flows observed by market makers are affected by their beliefs about I, U, and X, so they must first figure out the trading strategy of the insider. We conjecture that the sufficient set of state variables at time t consists of I_t, U_t, and X_t, and market makers' conditional expectations of these variables, $\widehat{I}_t$, $\widehat{U}_t$, and $\widehat{X}_t$, where $\widehat{I}_t = E[I_t|\mathcal{F}_M(t)]$, $\widehat{U}_t = E[U_t|\mathcal{F}_M(t)]$, and $\widehat{X}_t = E[X_t|\mathcal{F}_M(t)]$. We confirm this conjecture later.

Lemma 1. $\widehat{U}_t$ and $\widehat{X}_t$ satisfy the condition:

$$(U_t - \widehat{U}_t) = -(X_t - \widehat{X}_t). \tag{9}$$

Proof. $E[\omega_t|\mathcal{F}_M(t)] = \omega_t = X_t + U_t = \widehat{X}_t + \widehat{U}_t$ or $(U_t - \widehat{U}_t) = -(X_t - \widehat{X}_t)$. ■

According to Lemma 1, given the order flow, if market makers increase their estimation of U_t, then they will lower their estimation of X_t accordingly. Conjecture that insider's order rate is a linear function of market makers' estimation errors of I, U, and X:

$$\theta_t = f_1(I_t - \widehat{I}_t) + f_2(U_t - \widehat{U}_t) + f_3 X_t, \tag{10}$$

where f_1, f_2, and f_3 are constants. When the noise supply follows a random walk and the insider is risk neutral, insider's trade reduces to $\theta_t = f_1(I_t - \widehat{I}_t)$, as in CV's (2008) work. For simplicity, we shall omit subscript t in the rest of the paper whenever it does not cause confusion. We next summarize the solutions to market makers' filtering problem.

Proposition 1. Market makers' beliefs, $(\widehat{I}, \widehat{U}, \widehat{X})^T$, evolve according to

$$\begin{pmatrix} d\widehat{I} \\ d\widehat{U} \\ d\widehat{X} \end{pmatrix} = \begin{bmatrix} -\alpha_I(1+m_1) & ah_1 & -f_3h_1 \\ -m_2\alpha_I & -a(1-h_2) & -f_3h_2 \\ -m_3\alpha_I & ah_3 & f_3(1-h_3) \end{bmatrix} \begin{pmatrix} \widehat{I} \\ \widehat{U} \\ \widehat{X} \end{pmatrix} dt + \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} d\omega + \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} dD, \tag{11}$$

where the updating rules $h = (h_1, \quad h_2, \quad h_3)^T$ and $m = (m_1, \quad m_2, \quad m_3)^T$ are 3×1 vectors determined by

$$\begin{pmatrix} h_1 & m_1 \\ h_2 & m_2 \\ h_3 & m_3 \end{pmatrix} = \left[\Sigma_t \begin{pmatrix} f_1 & \alpha_I \\ f_2 - a & 0 \\ f_3 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\eta\sigma_D^2 \\ \sigma_U^2 & 0 \\ 0 & 0 \end{pmatrix} \right] \times \begin{pmatrix} 1/\sigma_U^2 & 0 \\ 0 & 1/\sigma_D^2 \end{pmatrix}. \tag{12}$$

The variance-covariance matrix

$$\Sigma_t = E\left[\begin{pmatrix} I-\widehat{I} \\ U-\widehat{U} \\ X-\widehat{X} \end{pmatrix}\left(I-\widehat{I},\quad U-\widehat{U},\quad X-\widehat{X}\right)|\mathcal{F}_M(t)\right] \tag{13}$$

is determined by

$$\Sigma_t = \Lambda^T \bar{\Sigma}_t \Lambda,\quad \Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix},\quad \bar{\Sigma}_t = \begin{pmatrix} \Sigma_{11}(t) & \Sigma_{12}(t) \\ \Sigma_{21}(t) & \Sigma_{22}(t) \end{pmatrix}. \tag{14}$$

$\bar{\Sigma}_t$ is given by

$$d\bar{\Sigma}_t = \bar{a}_0\bar{\Sigma} + \bar{\Sigma}\bar{a}_0^T + \bar{\Omega} - \left[\bar{\Sigma}q_0{}^T + q_1\right](q_s)^{-1}\left[\bar{\Sigma}q_0{}^T + q_1\right]^T, \tag{15}$$

where

$$\bar{\Omega} = \begin{pmatrix} 2\eta\sigma_D^2 & 0 \\ 0 & \sigma_U^2 \end{pmatrix},\quad q_0^T = \begin{pmatrix} f_1 & \alpha_I \\ f_2 - a - f_3 & 0 \end{pmatrix},$$

$$q_1 = \begin{pmatrix} 0 & -\eta\sigma_D^2 \\ \sigma_U^2 & 0 \end{pmatrix},\quad q_s = \begin{pmatrix} \sigma_U^2 & 0 \\ 0 & \sigma_D^2 \end{pmatrix},\quad \bar{a}_0 = \begin{pmatrix} -\alpha_I & 0 \\ 0 & -a \end{pmatrix}.$$

The steady-state equilibrium is defined by the condition that $d\Sigma_t = 0$. $d\Sigma_t = 0$ if and only if $\Sigma_t = \Sigma^* = \Lambda^T\bar{\Sigma}^*\Lambda$ for all time t, where $\bar{\Sigma}^*$ is a 2×2 matrix. In the steady-state equilibrium, $\bar{\Sigma}^*$ can also be expressed as a function of h and m:

$$2\alpha_I\Sigma_{11}^* = \left(2\eta\sigma_D^2 - h_1^2\sigma_U^2 - m_1^2\sigma_D^2\right),\quad (a+\alpha_I)\Sigma_{12}^* = -\left(h_1h_2\sigma_U^2 + m_1m_2\sigma_D^2\right),$$

$$2a\Sigma_{22}^* = \left[(1-h_2^2)\sigma_U^2 - m_2^2\sigma_D^2\right]. \tag{16}$$

As shown in Equation (11), market makers update their beliefs about I_t, U_t, and X_t given both their prior beliefs about I_{t-dt}, U_{t-dt}, and X_{t-dt} at time $t-dt$ and the new information at time t, including the order flow $d\omega_t$ and the dividend surprise dD_t. We term the constants h and m the updating rules of market makers with respect to the order flow and dividend surprise, respectively. Note that the drift part of $(\widehat{I},\widehat{U},\widehat{X})$ reflects market makers' prior beliefs about (I_t, U_t, X_t), adjusted by their prior beliefs about $d\omega_t$ and dD_t at time $t-dt$. Insider's trading generates an endogenous relation between private fundamental information and nonfundamental information based on market maker's information set. Market makers then use order flows and dividends to update their beliefs on U and X (nonfundamental information), leading to nonzero m_2 and m_3. Because the insider trades positively on his fundamental information advantage, market makers perceive that they underestimate private fundamental information given a buy order, leading to a positive h_1. Lemma 1 implies that $h_2+h_3=1$ and $m_2=-m_3$.

Equation (12) links the updating rules h and m with the steady-state variance-covariance matrix Σ. h and m can be understood as the regression coefficients when I_t, U_t, and X_t are regressed on $d\omega_t$ and dD_t, given market makers' prior beliefs at time $t-dt$. To further understand market makers' updating rules, we restate $\widehat{I}$, $\widehat{U}$, and $\widehat{X}$ in the following corollary.

Corollary 1. The process of $(\widehat{I},\widehat{U},\widehat{X})^T$ can also be expressed as

$$\begin{pmatrix} d\widehat{I} \\ d\widehat{U} \\ d\widehat{X} \end{pmatrix} = \begin{bmatrix} -\alpha_I & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & f_3 \end{bmatrix} \begin{pmatrix} \widehat{I} \\ \widehat{U} \\ \widehat{X} \end{pmatrix} dt + \begin{bmatrix} \alpha_I m_1 + h_1 f_1 & -(a+f_3-f_2)h_1 \\ \alpha_I m_2 + h_2 f_1 & -(a+f_3-f_2)h_2 \\ \alpha_I m_3 + h_3 f_1 & -(a+f_3-f_2)h_3 \end{bmatrix}$$
$$\times \begin{pmatrix} I-\widehat{I} \\ U-\widehat{U} \end{pmatrix} dt + \begin{bmatrix} m_1\sigma_D & 0 & h_1\sigma_U \\ m_2\sigma_D & 0 & h_2\sigma_U \\ m_3\sigma_D & 0 & h_3\sigma_U \end{bmatrix} \begin{pmatrix} dB_1 \\ dB_2 \\ dB_3 \end{pmatrix}.$$

The drift parts of $d\widehat{I}$, $d\widehat{U}$, and $d\widehat{X}$ can be decomposed into two components. The first component is related to the mean-reverting speeds of I, U, and X, and the second component is related to market markers' estimation errors of I and U. As $U-\widehat{U}=-(X-\widehat{X})$, we do not need to include $X-\widehat{X}$ in the above process. Interestingly, market makers' updated beliefs depend on both $(I-\widehat{I})$ and $(U-\widehat{U})$, even though U is uncorrelated with I. The reason is that the insider trades on his fundamental and nonfundamental information advantages. Consequently, both I and U are incorporated into $\widehat{I}$. In contrast, when $a=0$ and $\gamma=0$, as in CV (2008), $f_2=f_3=0$ and $\widehat{I}$ is given by

$$d\widehat{I}=-\alpha_I\widehat{I}+(\alpha_I m_1+h_1 f_1)(I-\widehat{I})dt+m_1\sigma_D dB_1+h_1\sigma_U dB_3,$$

where $\widehat{I}$ does not depend on U.

2.2 Insider's investment opportunities

To characterize insider's investment opportunities, we define the instantaneous excess dollar return of the stock as $dQ=(D-rP)dt+dP$, which is the return on the zero-cost strategy of buying one share of stock by borrowing fully at the risk-free rate. Given the process of $(\widehat{I},\widehat{U})$ in Equation (11) and the conjectured trading strategy $dX=\left[f_1(I-\widehat{I})+f_2(U-\widehat{U})+f_3X\right]dt$, it is easy to calculate Q, $(I-\widehat{I})$, $(U-\widehat{U})$, and X. The results are summarized below.

Proposition 2. Let $Y^T=(I-\widehat{I},U-\widehat{U},X)$, where 'T' denotes the transpose. The process Y is of the following form:

$$dY\equiv a_Y Y dt+\zeta\theta dt+b_Y dB$$
$$=\begin{bmatrix} -(1+m_1)\alpha_I & (a+f_3)h_1 & f_3h_1 \\ -m_2\alpha_I & -a+(a+f_3)h_2 & f_3h_2 \\ 0 & 0 & 0 \end{bmatrix} Y dt+\begin{bmatrix} -h_1 \\ -h_2 \\ 1 \end{bmatrix}\theta dt$$
$$+\begin{bmatrix} -(\eta\sigma_D+m_1\sigma_D) & \sqrt{2\eta-\eta^2}\sigma_D & -h_1\sigma_U \\ -m_2\sigma_D & 0 & (1-h_2)\sigma_U \\ 0 & 0 & 0 \end{bmatrix} dB, \qquad (17)$$

where $dB=\left(dB_1,\quad dB_2,\quad dB_3\right)^T$. The excess return dQ is given by

$$dQ=dP+(D-rP)dt=a_Q^T Y dt+\lambda\theta dt+b_Q^T dB, \qquad (18)$$

where $\lambda \equiv \mu h_1$ measures the price impact, and a_Q and b_Q are functions of m, h, and f:

$$a_Q = \left[\tfrac{\alpha_I}{r} + \mu m_1 \alpha_I, \quad -\lambda(a + f_3), \quad -\lambda f_3\right]^T, \qquad b_Q = \left(\tfrac{\sigma_D}{r} + \mu m_1 \sigma_D, \quad 0, \quad \lambda \sigma_U\right)^T.$$

This proposition is a direct result of Proposition 1, and thus we omit its proof. $(I - \widehat{I}, U - \widehat{U}, X)$ are the state variables governing insider's investment opportunities. Note that $(a_Q^T Y + \lambda\theta)dt$ represents insider's expected return. This proposition demonstrates how insider's trading influences the expected values of dY and dQ. One unit increase in θ causes h_1 and h_2 unit decreases in $I - \widehat{I}$ and $U - \widehat{U}$, and λ dollar increases in Q. As λ is positive, Equation (18) illustrates that the insider perceives his buy order to increase the expected return. As in Kyle's (1985) work, λ measures the price impact, which is defined as the inverse of the order flow necessary to induce the price to rise or fall by one dollar, that is, $\lambda \equiv \frac{\partial dP}{\partial d\omega} = \mu \frac{\partial d\widehat{I}}{\partial d\omega} = \mu h_1$, and market depth is defined as the reciprocal of λ.

2.3 Insider's maximization problem

Let W_t denote insider's wealth, θ_t his order rate in the stock, X_t his inventory in the stock, and C_t his consumption rate at time t. The insider's optimization problem is given by

$$\max_{\{\theta, C\}} E\left[-\int_t^\infty \frac{1}{\gamma}\exp(-\rho s - \gamma C_s)ds \,|\, \mathcal{F}_I(t)\right],$$

$$s.t. \quad dW = (rW - C - \frac{1}{2}k\theta^2)dt + X\,dQ. \tag{19}$$

Note that the control variable of the insider at time t is θ_t, whereas X_t is a state variable.

Let $J(W, Y, t)$ be the value function, which satisfies the following Bellman equation:

$$\begin{aligned} 0 \;=\; & \max_{\{\theta, C\}} E\left[-\frac{1}{\gamma}\exp(-\rho t - \gamma C_t)dt + dJ(W, Y, t) \,|\, \mathcal{F}_I(t)\right], \\ & s.t. \quad dW = (rW - C - \frac{1}{2}k\theta^2)dt + X\,dQ, \\ & \lim_{s \to \infty} E[J(W, Y, t+s) \,|\, \mathcal{F}_I(t)] = 0, \end{aligned} \tag{20}$$

where $\mathcal{F}_I(t) \equiv \{(D_s, I_s, U_s) : s \in (-\infty, t]\}$. The solution to this problem is given below.

Proposition 3. The optimal value function of the insider is given by

$$J(W, Y, t) = -\frac{1}{r\gamma}\exp\left[-\rho t - Z_0 - \gamma\left(rW + V_0 + \frac{Y^T L Y}{2\gamma}\right)\right], \tag{21}$$

where $Z_0 = -(r-\rho)/r$, and $V_0 = \frac{1}{2r\gamma} tr(b_Y^T L b_Y)$ represents the annualized steady-state value of private fundamental information. The optimal order rate θ is given by

$$\theta = \frac{r\gamma\lambda X + Y^T L\zeta}{r\gamma k}. \tag{22}$$

The optimal consumption rate is given by

$$C = \frac{1}{\gamma}\left[r\gamma W + \gamma V_0 + Z_0 + \frac{1}{2}Y^T L Y\right], \tag{23}$$

where L satisfies the following equation at steady state:

$$\begin{aligned} 0 = {} & rL - r\gamma(i_3 a_Q^T) - r\gamma(a_Q i_3^T) - (a_Y^T L + L a_Y) + (r\gamma)^2 (b_Q^T b_Q)(i_3 i_3^T) \\ & + L b_Y b_Y^T L - r\gamma k\left(\frac{L\zeta}{r\gamma k} + \frac{\lambda i_3}{k}\right)\left(\frac{L\zeta}{r\gamma k} + \frac{\lambda i_3}{k}\right)^T + (r\gamma) L b_Y b_Q i_3^T \\ & + (r\gamma) i_3 b_Q^T b_Y^T L, \end{aligned} \tag{24}$$

where $i_3 = \begin{pmatrix} 0, & 0, & 1 \end{pmatrix}^T$, and a_Q, b_Q, a_Y, ζ, and b_Y are defined in Proposition 2.

Equation (22) verifies the conjectured form of insider's trading strategy. The insider's order rate depends on $I - \widehat{I}, U - \widehat{U}$, and X. Notice that insider's trading strategy depends only on $I - \widehat{I}$ in Kyle (1985), Back (1992), and CV (2008). We now explain the implication of this strategy. The insider trades positively on his fundamental information advantage, $I - \widehat{I}$, so $f_1 > 0$. Because the insider is risk averse, he tends to hedge the inventory risk by trading against his inventory ($f_3 < 0$); that is, he reduces (increases) his inventory when it is high (low). The risk-averse insider also reduces his trading on fundamental information compared with a risk-neutral insider (a smaller f_1), as those of Subrahmanyam (1991) and Baruch (2002). We term this effect the *hedging effect*.

We now discuss the intuition of insider's trading on $U - \widehat{U}$. Because the insider observes the price, he can infer U perfectly. The insider can then estimate future order flows more accurately than can market makers, because of the mean-reverting properties of U and X. As a result, the insider can trade on both $U - \widehat{U}$ and $X - \widehat{X}$, which is equivalent to trading only on $U - \widehat{U}$.[16] We refer to insider's nonfundamental information advantage as $U - \widehat{U}$.

Importantly, there exists a self-reinforcing mechanism between insider's trading on fundamental information and that on nonfundamental information, which we term the *feedback effect*. Insider's trading on nonfundamental information makes it more difficult for market makers to infer fundamental

[16] Lemma 1 shows that $U - \widehat{U} = -(X - \widehat{X})$.

information, allowing more aggressive trading by the insider on fundamental information than in the case in which the insider has no advantage on nonfundamental information.[17] When the feedback effect is sufficiently strong, we demonstrate that a risk-averse insider can trade even more aggressively on fundamental information (larger f_1) than a risk-neutral one does. The next section shows that the feedback effect is the key to understand the results on price informativeness, liquidity, and price volatility.

The insider in our model trades strategically and smoothes his order over time. Using Equations (9) and (10), we obtain that based on the information set of market makers, observing ω_t is equivalent to observing

$$U_t + \int_{-\infty}^{t} [f_I I_s + (f_2 - f_3) U_s] ds. \tag{25}$$

Notice that this variable is a linear function of the noise supply U and the expected growth rate of dividends I in both the current period and the previous periods. This historical dependency enables the insider to trade on both his fundamental and his nonfundamental information advantages over market makers separately. In contrast, the competitive informed investors of Wang (1993) trade only on their information advantage on the fundamental information, and trading on their information advantage regarding the noise supply does not add value to them. Intuitively, conditional on the uninformed traders' information set, observing the price is equivalent to observing a linear function of the noise supply U and the expected growth rate of dividends I in the current period. Consequently, according to Lemma 4.1 of Wang (1993), the informed investors' information advantages on I and U are perfectly negatively correlated.

We next examine the limiting case in which the insider is risk neutral ($\gamma = 0$). We assume that $\rho = r$, because if $\rho \neq r$, a risk-neutral insider either consumes everything instantaneously or postpones consumption permanently.

Corollary 2. When the insider is risk neutral, the solution to Equation (20) is given by

$$J(W, Y, t) = \frac{1}{r} \exp(-rt) \left(rW + \bar{V}_0 + \frac{1}{2} Y^T \bar{L} Y \right). \tag{26}$$

The optimal order rate θ is given by

$$\theta = \frac{r\lambda X + Y^T \bar{L} \zeta}{rk}. \tag{27}$$

The optimal consumption rate is given by

$$C = rW + \bar{V}_0 + \frac{1}{2} Y^T \bar{L} Y, \tag{28}$$

[17] Note that insider's trading on fundamental information makes it more difficult for market makers to infer nonfundamental information as well, allowing more aggressive trading by the insider on nonfundamental information.

where $\bar{V}_0 = \frac{1}{2r} tr(b_Y^T \bar{L} b_Y)$ and $\bar{L}$ satisfies the following equation at steady state:

$$0 = r\bar{L} - r(i_3 a_Q^T) - r(a_Q i_3^T) - (a_Y^T \bar{L} + \bar{L} a_Y) - rk\left(\frac{\bar{L}\zeta}{rk} + \frac{\lambda i_3}{k}\right)\left(\frac{\bar{L}\zeta}{rk} + \frac{\lambda i_3}{k}\right)^T. \quad (29)$$

This corollary is used to examine the equilibrium properties in the next section when the insider is risk neutral, in which only the feedback effect exists.

Equations (12), (16), (22), and (24) determine the equilibrium. We can then solve insider's consumption from Equation (23). When $\gamma = 0$, we can solve the equilibrium more conveniently with Equations (12), (16), (27), and (29).

3. Price Informativeness, Market Liquidity, and Price Variability

In this section, we present our main results regarding the equilibrium properties, such as the price informativeness, market liquidity, and price variability. For comparison with Vayanos (2001) and CV (2008), we focus on the limiting case in which the quadratic cost coefficient approaches zero. Some of our main results, such as the price is not fully revealing, will obviously go through when the cost coefficient is a positive constant.

We define the price informativeness by $RI \equiv 1 - \frac{Var(I_t|\mathcal{F}_M(t))}{Var(I_t|D_s, s\in(-\infty,t])}$, that is, the proportion of private fundamental information I_t being incorporated into the price due to the insider trading at time t. Because we study only the steady-state equilibrium, RI is constant over time and is in the range between zero and one. The constant price informativeness implies that new information is incorporated into the price at a constant rate. Using Equation (16), a simple calculation shows that $Var(I_t|\mathcal{F}_M(t)) = \left[2\eta\sigma_D^2 - h_1^2\sigma_U^2 - m_1^2\sigma_D^2\right]/(2\alpha_I)$ and $RI = \left[h_1^2\sigma_U^2 + m_1^2\sigma_D^2\right]/(2\eta\sigma_D^2)$.

Similar to Kyle (1985), we measure the stock price variability by the instantaneous variance of the price. Differentiating the stock price process in Equation (8) yields

$$dP_t = \frac{1}{r} dD_t + \mu d\widehat{I}_t. \quad (30)$$

The instantaneous variance of the price is given below.

Proposition 4. The instantaneous variance rate of the price is of the form:

$$\sigma_P^2 = Var[dP|\mathcal{F}_M]/dt = \left[\frac{\sigma_D^2}{r^2} + 2\mu^2(\eta\sigma_D^2 - \Sigma_{11}\alpha_I) + \frac{2m_1\mu\sigma_D^2}{r}\right]. \quad (31)$$

The variance comprises three parts. The first part $\sigma_{P_D}^2 \equiv \sigma_D^2/r^2$ is the instantaneous variance of the present value of future dividends based on D; the second part $2\mu^2(\eta\sigma_D^2 - \Sigma_{11}\alpha_I)$ represents the instantaneous variance of the present value of I_t based on D and ω; and the third part $2m_1\mu\sigma_D^2/r$ represents the adjustment due to the correlation between dD and dI.

3.1 Fully revealing equilibrium with a risk-neutral insider

There are two ways of studying the equilibrium properties in the continuous-time limit with a risk-neutral insider. CV (2008) start with a discrete-time setting in which there is no trading cost. They arrive at the continuous-time limit by taking the time interval to be zero. We start with a continuous-time model directly and then take the quadratic cost to be zero. To obtain compatible results, we impose that $a=0$ and $\gamma=0$ as in CV, namely, the noise supply follows a random walk process and the insider is risk neutral.

Proposition 5. When $a=0$, $\gamma=0$, and the quadratic cost is small ($k\to 0$), the limiting behaviors of the equilibrium are given by

$$h_1 \to \widetilde{h}_1 = \frac{\sqrt{2\eta-\eta^2}\sigma_D}{\sigma_U}, \quad m_1 \to \widetilde{m}_1 = -\eta, \tag{32}$$

$$\sqrt{k} f_1 \to \widetilde{f}_1 = \sqrt{\frac{\mu[r+2(1-\eta)\alpha_I]}{\widetilde{h}_1}}, \quad f_2 = 0, \quad f_3 = 0, \tag{33}$$

$$\bar{L} \to \begin{pmatrix} \frac{r\widetilde{f}_1^{\,2}}{r+2(1-\eta)\alpha_I} & 0 & r\mu \\ 0 & 0 & 0 \\ r\mu & 0 & 0 \end{pmatrix}, \quad \bar{V}_0 \to \mu\sqrt{(2\eta-\eta^2)}\sigma_D\sigma_U, \tag{34}$$

$$\frac{\Sigma_{11}}{\sqrt{k}} \to \widetilde{\Sigma_{11}} = \frac{\widetilde{h}_1\sigma_U^2}{\widetilde{f}_1}, \tag{35}$$

where $\bar{L}$ and $\bar{V}_0$ are defined in Corollary 2.

This proposition shows that Σ_{11}, the conditional variance of market makers' estimation error of fundamental information, $(I_t-\widehat{I}_t)$, goes to zero as k approaches zero, leading to a fully revealing equilibrium in the limit. Note that because the insider reveals perfectly his private information, market makers can infer the values of dB_1 and dB_2. Market makers know that the shock to I is $-\eta\sigma_D dB_1+\sqrt{2\eta-\eta^2}\sigma_D dB_2$, they can deduct the shock related to dB_1 using the dividend surprise $\eta\sigma_D dB_1$. Hence, m_1 converges to $-\eta$. As a result, h_1 converges to a constant $\widetilde{h}_1$ and depends on only the instantaneous variance rate of the dB_2 part in the dI process. The insider's trading intensity on the fundamental information, f_1, goes to infinity in the order of $1/\sqrt{k}$. Because the insider trades only on his fundamental informational advantage, f_2 and f_3 are both zero, and market makers need to update only their beliefs regarding I. Hence, the expressions related to the nonfundamental information, such as Σ_{12}, Σ_{22}, h_2, m_2, are absent.

By rearrangement, insider's value function has a simple form in the limit:

$$J(W,Y,t)=\frac{1}{r}\exp(-rt)\left[rM+rXV+\mu\sqrt{(2\eta-\eta^2)}\sigma_D\sigma_U\right],$$

where V is the fundamental value defined in Equation (4). The value function is determined by two parts. First, $M+XV=M+(D/r+\mu I)X$ represents insider's wealth with the stock evaluated at the fundamental value V, where M denotes insider's position in the bond. The second part represents the value of his private information. We obtain a positive value of private information, because the insider trades infinitely aggressively though his profit per share goes to zero.

Intuitively, the steady-state equilibriums of Vayanos (2001), CV (2008), and this special case of our model ensure that the price impact is constant. The insider is impatient because the value of old information decays with new information arriving continuously and he discounts future consumption at a rate of r. Hence, even a risk-averse insider (as that of Vayanos (2001)) tends to trade infinitely aggressively on market makers' estimation error of the private information in the continuous-time limit, as he can walk along the residual supply curve infinitely fast. Consequently, the insider reveals his private information instantaneously. We next demonstrate that the price no longer reveals private information instantaneously when the insider also trades on his advantage over nonfundamental information.

3.2 Nonfully revealing equilibrium with a mean-reverting noise supply and a risk-neutral insider

Here, we consider the scenario in which insider's trading on nonfundamental information is caused by the overreaction of noise traders or the mean reversion of the noise supply (a). We impose that the insider is risk neutral ($\gamma=0$). In this case, the equilibrium is driven only by the feedback effect. We focus on the case in which the trading cost approaches zero. To obtain semi-closed-form solutions, we consider a special case in which a is small.[18] We then use numerical solutions to study the case of a general a when k is small. We summarize the results in the following proposition.

Proposition 6. When $\gamma=0$, $a\to 0$, and $k\to 0$, there exists a linear, nonfully revealing equilibrium, and the limiting properties are given by

$$\Sigma_{11}\to\sqrt{k}\widetilde{\Sigma_{11}}+a\underline{\Sigma_{11}},\quad \Sigma_{12}\to\underline{\Sigma_{12}},\quad \Sigma_{22}\to\frac{\widetilde{\Sigma_{22}}}{a}, \tag{36}$$

$$f_1\to\frac{\widetilde{f_1}+a\underline{f_1}}{\sqrt{k}},\quad f_2\to\frac{a\underline{f_2}}{\sqrt{k}}, \tag{37}$$

$$h_1\to\widetilde{h_1}+a\underline{h_1},\quad h_2\to\widetilde{h_2},\quad m_1\to\widetilde{m_1}+a\underline{m_1},\quad m_2\to\widetilde{m_2}, \tag{38}$$

$$\frac{\underline{\Sigma_{11}}}{\underline{\Sigma_{12}}}=\frac{\Sigma_{12}}{\widetilde{\Sigma_{22}}}\to-\frac{f_2}{\widetilde{f_1}}. \tag{39}$$

[18] CK (1993) find empirically that the mean-reverting speed of the noise supply is small.

where $\widetilde{h_1} = \frac{\sqrt{2\eta-\eta^2}\sigma_D}{\sigma_U}$, $\widetilde{m_1} = -\eta$, $\widetilde{f_1} = \sqrt{\frac{\mu[r+2(1-\eta)\alpha_I]}{\widetilde{h_1}}}$, $\widetilde{\Sigma_{11}} = \frac{\widetilde{h_1}\sigma_U^2}{\widetilde{f_1}}$, $\widetilde{\Sigma_{22}} > 0$, $\widetilde{h_2} > 0$, $\underline{f_1} > 0$, $\underline{f_2} < 0$, $\underline{\Sigma_{11}} > 0$, and $\underline{m_1} > 0$. Also, $\underline{h_1} < 0$, if $\eta < 1$; $\underline{h_1} = 0$, if $\eta = 1$; $\underline{h_1} > 0$, if $\eta > 1$. The instantaneous variance rate of the price σ_p^2 is given by

$$\sigma_P^2 \to \sigma_{p_f}^2 + \left(\frac{2\mu\sigma_D^2\underline{m_1}}{r+\alpha_I}\right)a,$$

where $\sigma_{p_f}^2 = \frac{\sigma_D^2}{r^2} - 2\mu^2\frac{r}{\alpha_I}\eta\sigma_D^2$ is the fully revealing price variability.

Note that the expressions for $\widetilde{\Sigma_{11}}$, $\widetilde{h_1}$, $\widetilde{m_1}$, and $\widetilde{f_1}$ take the same form as in the benchmark case in which the insider is risk neutral and the noise supply follows a random walk process. Because the insider does not trade on nonfundamental information in the benchmark case, $\underline{\Sigma_{11}}$, $\underline{h_1}$, $\underline{m_1}$, $\underline{f_1}$, and $\underline{f_2}$ represent the deviations of the solutions from those in the benchmark case. The insider now trades on his information advantages on both fundamental information and the noise supply, so there is an endogenous correlation between these two types of information, that is, Σ_{12}, converges to a nonzero constant $\underline{\Sigma_{12}}$. In addition, the insider uses dividend and order flow information to update his beliefs on the noise supply; that is, h_2 and m_2 converge to two constants. Σ_{22}, the conditional variance of market makers' estimation error of the noise supply, also converges to a finite constant $\widetilde{\Sigma_{22}}/a$.[19]

This proposition shows that as k approaches zero, f_1 and f_2 approach infinity in the same order of $1/\sqrt{k}$,[20] but Σ_{11}, the conditional variance of market makers' estimation error of the fundamental information, remains positive as long as a is nonzero. We thus arrive at an equilibrium price that is not fully revealing.

The feedback effect between insider's trading on his informational advantage over the noise supply and his trading on his informational advantage over fundamental information explains the nonfully revealing result. Though the insider does not observe noise directly, as he knows his own inventory, he can back out perfectly the noise supply from the price. Because the noise is mean reverting, knowing the noise supply in the current period gives the insider information advantage over market makers regarding the future realizations of the noise supply. For example, when market makers underestimate the noise supply in the current period, they tend to overestimate the noise supply in future periods, inducing the insider to trade negatively on $U - \widehat{U}$ ($f_2 < 0$). This in turn allows the insider to camouflage his fundamental information by trading on his nonfundamental information, leading to more aggressive trading on $I - \widehat{I}$ than

[19] We omit the discussion related to market makers' beliefs on insider's inventory X, which can be derived from Proposition 1.

[20] This is a key result that leads to a nonfully revealing price. If f_1 were infinity but f_2 were finite, then the price would also have been fully revealing. Even if f_1 were to approach infinity faster than f_2, the price would have been fully revealing.

in the benchmark case of a random noise ($\underline{f_1} > 0$). Strikingly, Equation (39) shows that as the trading cost goes to zero, market makers' estimation errors, $I - \widehat{I}$ and $U - \widehat{U}$, tend to be perfectly correlated. This result is due to insider's infinitely aggressive trading on these two types of information.[21] It means that market makers cannot make any distinction between insider's trading on his two types of informational advantages when the trading cost goes to zero, leading to a nonfully revealing equilibrium.

Note that when the noise supply follows a random walk, as in Kyle's (1985) and CV's (2008) work, the insider can also infer the noise supply in the previous period from the price. Because noise is random, knowing previous noise does not give the insider information advantage on future noise. Hence, the insider will not trade on noise.

This proposition also shows that because of the feedback effect, the price is more volatile than in the case in which the noise supply follows a random walk process. In other words, through the feedback effect, the overreaction of noise traders contributes to the "excess price volatility" of a stock compared with the simple present value model with a constant discount rate (see Campbell and Shiller, 1988; Fama and French, 1988). When the correction speed of noise traders' overreaction increases (an increase in the mean-reverting speed of the noise supply), because of the feedback effect, the insider trades more aggressively on both nonfundamental and fundamental information, leading to a less informative and more volatile price. Economically, we demonstrate that rational traders can destabilize prices in the presence of overreacting noise traders. In contrast, in CK's (1993) and Wang's (1993) work, noise trading does contribute to the excess price volatility, but when the correction speed of noise traders' overreaction increases, the price generally becomes less volatile. When the noise supply reverts to its long-term value faster, the amount of noise risk declines, so rational investors would require a smaller risk premium in providing liquidity to noise traders, leading to a smaller price variability.

Because of the feedback effect, our model also generates interesting results on the relationship between the mean-reverting speed of the noise supply a and the price impact λ.

Corollary 3. Suppose $a \to 0$ and $k \to 0$. If the amount of private fundamental information is small ($\eta < 1$), then λ declines as a increases; if the amount of private fundamental information is large ($\eta > 1$), then λ rises as a increases.

Recall that $\lambda = \mu h_1$. When the amount of private information is small (a small η), the insider has less incentive to camouflage his private fundamental

[21] This perfect correlation can be seen from Equation (12). As the trading cost k goes to zero, both f_1 and f_2 converge to infinity. As in Proposition 6, denote that $\underline{f_1} = \tilde{f}_1/\sqrt{k}$ and $\underline{f_2} = \tilde{f}_2/\sqrt{k}$. Because both h_1 and h_2 converge to finite numbers, we obtain that $\Sigma_{11}\tilde{f}_1 + \Sigma_{12}\tilde{f}_2 \to 0$ and $\Sigma_{12}\tilde{f}_1 + \Sigma_{22}\tilde{f}_2 \to 0$. Rearrangement yields that $\frac{\Sigma_{11}}{\Sigma_{12}} - \frac{\Sigma_{12}}{\Sigma_{22}} \to 0$, which means that the correlation between $I - \widehat{I}$ and $U - \widehat{U}$ converges to 1 or -1.

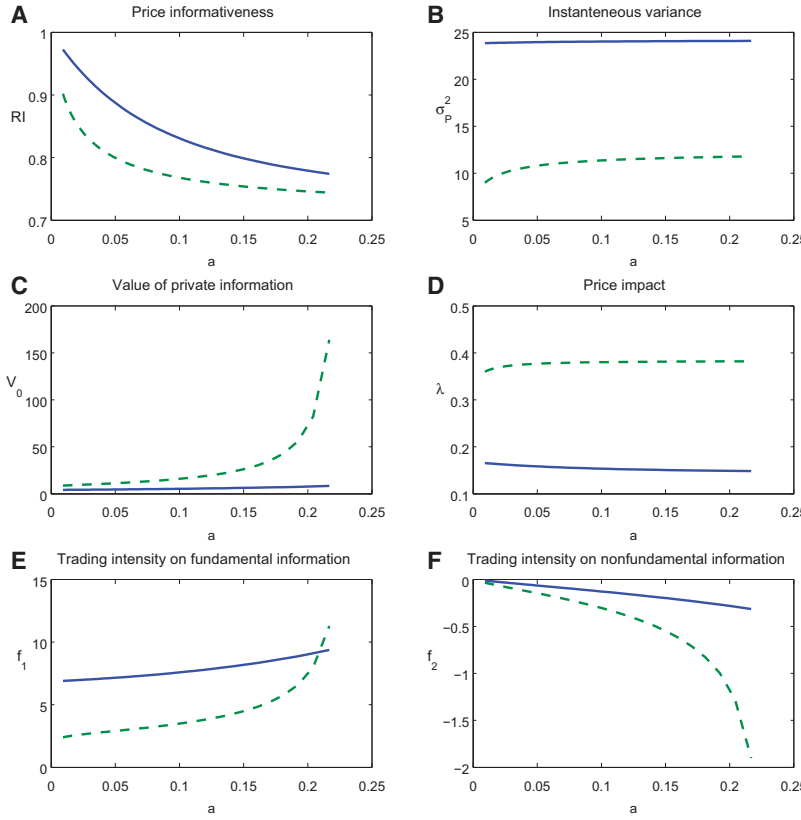

Figure 1
Effects of the mean-reversion coefficient of noise supply Panels A–F plot price informativeness, price variability, value of private information, price impact, insider's trading intensity on fundamental information, and insider's trading intensity on noise supply. We set $r=0.04$, $\gamma=0$, $\sigma_D=0.2$, $\sigma_U=5$, and $\alpha_I=0.025$. The solid and dashed lines correspond to $\eta=0.1$ and $\eta=1.5$, respectively.

information by his trading on nonfundamental information. As a increases, the increase in his trading aggressiveness on fundamental information is relatively small, leading to decreases in h_1 and λ. Hence, the feedback effect is weak. When η is large, the insider has more incentive to camouflage his private fundamental information by his trading on nonfundamental information. As a increases, the increase in his trading aggressiveness on fundamental information is relatively large, leading to increases in h_1 and λ. In this case, the feedback effect is strong.

To demonstrate the robustness of both the semi-closed-form solutions and our logic, we also present the results for a general a using numerical solutions. Figure 1 shows the comparative statics regarding a for $\eta=0.1$ and $\eta=1.5$, respectively. It illustrates that because of the feedback effect, an increase in a leads to increases in the magnitudes of both f_1 and f_2. A rise in a leads to a

less informative and more volatile price,[22] and a larger expected profit for the insider. In addition, a rise in a leads to a smaller price impact for $\eta=0.1$ but a larger price impact for $\eta=1.5$.[23]

3.3 Nonfully revealing equilibrium with a risk-averse insider

We consider the scenario in which insider's trading on nonfundamental information is caused by the risk aversion of the insider. The equilibrium is now more complicated because insider's trading is driven by both the feedback effect and the hedging effect, induced by insider's risk aversion. We first study a special case in which both insider's risk aversion (γ) and his trading cost coefficient (k) approach zero from above. We also impose that the noise supply follows a random walk as in CV's model ($a=0$). Using numerical calculations, we then obtain the results for a general γ when k is small. We summarize the semi-closed-form solutions as follows.

Proposition 7. When $a=0$, $\gamma\to 0$, and $k\to 0$, there exists a linear, nonfully revealing equilibrium, and the limiting properties are given by

$$\Sigma_{11}\to\sqrt{k}\widetilde{\Sigma_{11}}+\gamma\underline{\Sigma_{11}},\quad \Sigma_{12}\to\underline{\Sigma_{12}},\quad \Sigma_{22}\to\frac{\widetilde{\Sigma_{22}}}{\gamma}, \tag{40}$$

$$f_1\to\frac{\widetilde{f_1}+\gamma\underline{f_1}}{\sqrt{k}},\quad f_2\to\frac{\gamma\underline{f_2}}{\sqrt{k}},\quad f_3\to\gamma\underline{f_3}, \tag{41}$$

$$h_1\to\widetilde{h_1}+\gamma\underline{h_1},\quad h_2\to\widetilde{h_2},\quad m_1\to\widetilde{m_1}+\gamma\underline{m_1},\quad m_2\to\widetilde{m_2}, \tag{42}$$

$$\frac{\Sigma_{11}}{\Sigma_{12}}=\frac{\Sigma_{12}}{\widetilde{\Sigma_{22}}}\to-\frac{f_2}{\widetilde{f_1}}. \tag{43}$$

where $\widetilde{h_1}=\frac{\sqrt{2\eta-\eta^2}\sigma_D}{\sigma_U}$, $\widetilde{m_1}=-\eta$, $\widetilde{f_1}=\sqrt{\frac{\mu[r+2(1-\eta)\alpha_I]}{\widetilde{h_1}}}$, $\widetilde{\Sigma_{11}}=\frac{\widetilde{h_1}\sigma_U^2}{\widetilde{f_1}}$, $\underline{\Sigma_{11}}>0$, $\underline{m_1}>0$, $\underline{f_2}>0$, $\underline{f_3}<0$, and $\widetilde{h_2}>0$. Also, $\underline{h_1}<0$ if $\eta<1$; $\underline{h_1}=0$ if $\eta=1$; $\underline{h_1}>0$ if $\eta>1$. The annualized steady-state value of private information is given by

$$V_0\to\widetilde{V_0}+\underline{V_0}\gamma=\mu\sqrt{(2\eta-\eta^2)}\sigma_D\sigma_U+\left(\frac{\underline{L_{33}}\sigma_U^2}{r}\right)\gamma, \tag{44}$$

where $\underline{L_{33}}<0$. The instantaneous variance rate of the price is given by

$$\sigma_P^2\to\sigma_{p_f}^2+\left(\frac{2\mu\sigma_D^2\underline{m_1}}{r+\alpha_I}\right)\gamma,$$

where $\sigma_{p_f}^2=\frac{\sigma_D^2}{r^2}-2\mu^2\frac{r}{\alpha_I}\eta\sigma_D^2$ is the fully revealing price variability.

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[22] In other words, when the correction speed of noise traders' overreaction increases, the rational risk-neutral insider destabilizes the stock price, contributing to the excess price volatility.

[23] It is noteworthy that for general values of a, we find that when $\eta<\eta_0$, the price impact increases with a and when $\eta<\eta_0$, the price impact declines with a, where η_0 can differ from the threshold value of one in Corollary 3.

Note that the expressions for $\widetilde{\Sigma_{11}}$, $\widetilde{h_1}$, $\widetilde{m_1}$, and $\widetilde{f_1}$ take the same form as those in the benchmark case in which the insider is risk neutral and the noise supply follows a random walk process. $\underline{\Sigma_{11}}$, $\underline{h_1}$, $\underline{m_1}$, $\underline{f_1}$, $\underline{f_2}$, and $\underline{f_3}$ in the present case represent the deviations of the solutions from those in the benchmark case. Because the insider now trades on his advantages on both fundamental information and his own inventory (alternatively, the noise supply), there is an endogenous correlation between these two types of information; that is, Σ_{12}, converges to a nonzero constant $\underline{\Sigma_{12}}$. In addition, the insider uses dividend and order flow information to update his beliefs about the noise supply; that is, h_2 and m_2 converge to two constants. Σ_{22}, the conditional variance of market makers' estimation error of the noise supply, also converges to a finite constant $\widetilde{\Sigma_{22}}/\gamma$.[24]

We have shown that in the limiting case, in which the transaction cost and insider's risk aversion approach zero, the stock price is not fully revealing. Insider's trading intensities on $I-\widehat{I}$ and $U-\widehat{U}$, or f_1 and f_2, go to infinity in the same order of $1/\sqrt{k}$, but his trading intensity on his inventory, f_3, converges to a finite constant. The insider trades at a higher rate on market makers' estimation error of his inventory or his advantage over market makers on nonfundamental information (due to the feedback effect) than on his inventory itself (due to the hedging effect).

Because of the feedback effect, the insider camouflages his private fundamental information by trading on his nonfundamental information, and vice versa. Market makers cannot distinguish between insider's trading on these two types of private information. Equation (43) shows that as trading cost goes to zero, market makers' estimation errors of I_t and U_t tend to be perfectly correlated, because of insider's infinitely aggressive trading on these two types of information. Hence, the price converges to be nonfully revealing; that is, Σ_{11}, the conditional variance of market makers' estimation error of fundamental information, converges to a positive number. Although the stock price is not fully revealing both in our model and in Wang's (1993) model, the mechanisms are different. In Wang's (1993) work, the risk aversion prevents the informed traders from taking large positions to exploit uninformed traders' estimation errors. If the informed traders were risk neutral, however, the price would be fully revealing. In our model, because of the feedback effect, the insider's trading intensities on market makers' estimation errors regarding both the private information and the noise supply approach infinity in the same order. Because market makers cannot distinguish between the two types of information, the price is not fully revealing.

This proposition shows that the stock price is less informative as the insider becomes more risk averse. As a result, the price volatility increases with insider's risk aversion. Economically, this result means that the risk-averse

[24] Similar to Section 3.2, we omit the discussion related to market makers' beliefs regarding insider's inventory X, which can be easily derived from Proposition 1.

insider can destabilize prices in the presence of noise traders, as conjectured by Kyle (1985, page 1320). As $\underline{V}_0 < 0$, the value of the signal is less than that when the insider is risk neutral. Hence, risk aversion decreases the value of private information. In addition, our model represents the first continuous-time Kyle-type model in which the expected informed trade based on market makers' information is nonzero; that is, $E[\theta_t | \mathcal{F}_M(t)] = f_3 \widehat{X}_t \neq 0$, because of the hedging effect.[25]

Because of both the hedging effect and the feedback effect, our model also generates interesting results on the relationship between insider's risk aversion γ and price impact λ.

Corollary 4. If the amount of private fundamental information is small ($\eta < 1$), then λ declines as γ increases; if the amount is large ($\eta > 1$), then λ rises as γ increases.

The traditional view is demonstrated by Subrahmanyam (1991) and Baruch (2002); that is, the price impact (λ) always decreases with insider's risk aversion (γ). In our model, λ can increase with γ when the amount of fundamental information (η) is large and the insider is not very risk averse.[26] On the one hand, a higher γ leads to less aggressive trading by the insider on fundamental information because of the hedging effect. On the other hand, because of the feedback effect, a higher γ causes the insider to trade more aggressively on his information advantage regarding his future inventories, which can camouflage more effectively his private fundamental information.[27] As a result, the insider can trade more aggressively on his private fundamental information.

Recall that $\lambda = \mu h_1$. When η is small, the insider has less incentive to camouflage his trading on fundamental information by his trading on nonfundamental information, leading to a weak feedback effect. As the hedging effect dominates, h_1, λ, and insider's trading intensity on the fundamental information all decline ($\underline{h}_1 < 0$ and $\underline{f}_1 < 0$). Notice that insider's trading intensity on the fundamental information is smaller than that when the insider is risk neutral, because the hedging effect dominates. When η is large, the insider is more incentivized to camouflage his private fundamental information by trading on his nonfundamental information. As γ rises, insider's trading intensity on

[25] Baruch (2002) extends the Kyle model by incorporating a risk-averse insider. Because the insider knows exactly the value of the risky asset, he does not hedge his inventory.

[26] When the insider is very risk averse, the hedging effect dominates, because the inventory risk, which is a quadratic function of insider's inventory, plays the first-order effect. The price impact then declines with insider's risk aversion.

[27] A risk-averse insider hedges his inventory risk, so he reduces (increases) his inventory in the stock when his inventory is high (low), leading to a mean-reverting insider's inventory. Because a more risk-averse insider trades more aggressively against his inventory, his inventory reverts to its mean more quickly. Hence, the insider can forecast his own future inventory more accurately, or he gains a greater information advantage regarding his inventory over market makers. Consequently, a more risk-averse insider trades more aggressively on his information advantage regarding his future inventory.

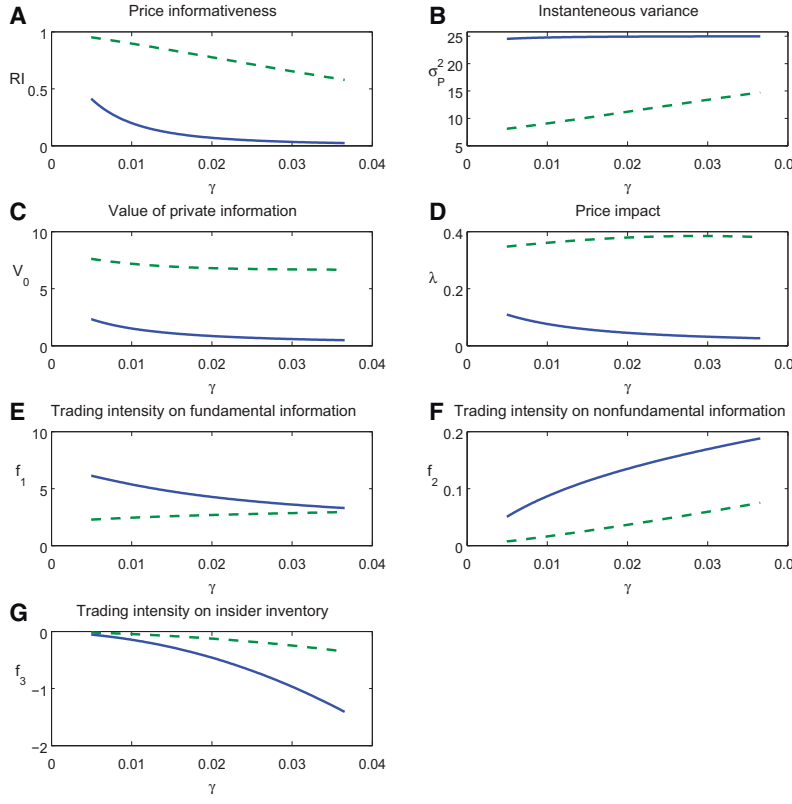

Figure 2
Effects of insider's risk aversion Panels A–G plot price informativeness, price variability, value of private information, price impact, insider's trading intensity on fundamental information, insider's trading intensity on noise supply, and insider's trading intensity on his inventory. We set $r=0.04$, $a=0$, $\sigma_D=0.2$, $\sigma_U=5$, and $\alpha_I=0.025$. The solid and dashed lines correspond to $\eta=0.1$ and $\eta=1.5$, respectively. For illustration purpose, we present $f_3\times 10$ for the case of $\eta=1.5$.

his fundamental information increases by a relatively large amount ($\underline{f_1}>0$), leading to increases in h_1 and λ ($\underline{h_1}>0$).[28] In this case, the feedback effect is strong. As a result, a risk-averse insider can trade more aggressively on his fundamental information than a risk-neutral one does.

To demonstrate the robustness of both our results and the logic, we next present the results for a general γ using numerical calculations. Figure 2 shows the comparative statics regarding γ for $\eta=0.1$ and $\eta=1.5$, respectively. It confirms that because of the feedback effect, an increase in γ leads to an increase in f_2 in both cases. Under both the hedging effect and the feedback effect, a rise in γ leads to a less informative and more volatile price, and a lower

[28] As the expressions for $\underline{f_1}$ and $\underline{f_2}$ are complicated, we are unable to determine their signs analytically, even though we have closed-form solutions. However, numerous of our calculations show that these results are valid.

value for the private information. In addition, a rise in γ leads to a decline in f_1 and a smaller price impact for a small η but a rise in f_1 and a larger price impact for a large η.[29]

To summarize, the results in Sections 3.2 and 3.3 illustrate the conditions under which the feedback effect escalates. When the risk aversion of the insider or the mean-reverting speed of the noise supply increases, the insider trades more aggressively on his nonfundamental information, camouflaging more effectively his private fundamental information. Hence, the insider trades more aggressively on his fundamental information. Because we are concerned about the impact of insider's trading on price efficiency and liquidity that are related directly to the fundamental value, we can measure the strength of the feedback effect by the part of insider's trading intensity on his fundamental information, which is induced by his trading on nonfundamental information. When the amount of private fundamental information is large, as the insider becomes more risk averse or the mean-reverting speed of the noise supply increases, he tends to trade relatively more aggressively on his fundamental information, leading to an escalated feedback effect and a larger price impact.

It is interesting to compare the two mechanisms associated with the mean reversion of the noise supply and the risk aversion of the insider, both of which induce the feedback effect that leads to nonfully revealing equilibriums. Because of the feedback effect, an increase in a or in γ causes the insider to trade more aggressively on $U-\widehat{U}$, leading to a less informative and more volatile price, as well as a smaller (larger) price impact when the amount of private information is small (large). The main difference is that the mean reversion in the noise supply generates a pure feedback effect, whereas the insider's risk aversion induces both a hedging effect and a feedback effect. As a result, for the risk-aversion mechanism, the value of private information is less than that when the insider is risk neutral and the noise supply follows a random walk, the expected insider's trade based on market makers' information is nonzero, and the insider can trade positively on $U-\widehat{U}$. The insider trades less intensively on fundamental information when the hedging effect dominates. When the noise supply follows a mean-reverting process and the insider is risk neutral, the value of private information is higher than that when the insider is risk neutral and the noise supply follows a random walk, the expected insider's trade based on market makers' information is zero, and the insider trades negatively on $U-\widehat{U}$. In addition, the insider trades more intensively on fundamental information than in the benchmark case of a random noise.

We have thus far focused on the limiting case in which the quadratic cost coefficient approaches zero. Using numerical calculations, we have also solved the equilibrium for general parameter values of insider's risk aversion, the mean-reverting speed of the noise supply process, and the cost function

[29] Notice that for general values of γ, we find that when $\eta > \eta_0$, the price impact increases with γ and when $\eta < \eta_0$, the price impact declines with γ, where η_0 can differ from the threshold value of one in Corollary 4.

coefficient. We have found that our main results are robust: the equilibrium price is not fully revealing, the price impact can increase with insider's risk aversion, and a more risk-averse insider can trade more aggressively on his private fundamental information than a less risk-averse one does. The numerical techniques and procedures are available from the authors upon request.

4. Empirical Implications

In this section, we explore the empirical implications of our model. These implications concern the impact of noise traders' overreaction on the idiosyncratic volatility of a stock, on insider's trading, and on the relationships about insider's current trade, his previous inventory, and future stock returns.

4.1 Overreaction of noise traders and feedback Effect

Since De Bondt and Thaler (1985, 1987), there is a large body of empirical literature on price overreaction. There are, however, few empirical studies on noise traders' overreaction. Our model yields rich and testable implications about the impact of the overreaction of noise traders on equilibrium properties, such as the volatility of individual stocks and the insider trading, which can be useful in guiding future empirical work.

The first implication is about the relationship between the correction speed of noise traders' overreaction and the idiosyncratic volatility.[30] Shiller (1981), Campbell and Shiller (1988) , and Fama and French (1988) show that stock prices are too volatile to be explained by the fundamentals. CK (1993) show theoretically that introducing noise trading is helpful to explain the "excess price volatility". They also provide supportive empirical evidence. The recent empirical studies of Andrade et al. (2008), Brandt et al. (2010), and Foucault et al. (2011) further demonstrate that retail trading (proxy for noise trading) contributes to price volatility.[31]

In the presence of noise traders' overreaction, the amount of noise trading is an increasing function of the instantaneous volatility of the noise supply (σ_u) but a decreasing function of the correction speed of noise traders' overreaction (a). The above studies, however, have not distinguished between the effects of σ_u and those of a on the price volatility. It is intuitive that a larger σ_u leads to a larger price volatility, but the effect of a is ambiguous. Controlling for σ_u, when a increases, CK predict that the price volatility declines because rational investors provide liquidity, whereas our model predicts that the price volatility increases, because of the feedback effect. Foucault et al. (2011) find that the decline in the magnitude of return reversals is associated with a drop in

[30] Because private information is more likely to be firm specific, we should use the idiosyncratic volatility in empirical tests and control for the price volatility due to systematic factors and industry factors.

[31] Barber et al. (2009) suggest that individual investors behave like noise traders.

the return volatility. According to the investor's overreaction hypothesis in De Bondt and Thaler (1985, 1987), the magnitude of return reversals increases with a. Hence, the empirical evidence in Foucault et al. (2011) may imply that the return volatility is positively correlated with a, consistent with the prediction of our model (see proposition 6).

Cross-sectionally, controlling for other factors, our model predicts that the idiosyncratic volatility of individual firms should increase with the correction speed of noise traders' overreaction. Following Barber et al. (2009), we can proxy noise traders by individual investors. We can run a time-series regression to estimate the mean-reverting speed of the aggregate position of individual investors of each firm as a proxy for the correction speed of noise traders' overreaction. We can then run cross-sectional regressions to test this prediction.

The second implication is about the insider trading. When studying insider's trading behavior, insider's trade is typically regressed on his private fundamental information, which is usually proxied by future returns or earnings, as in Kallunki, Nilsson. and Hellstrom (2009) work. Our model predicts that a faster correction speed of noise traders' overreaction (a larger a) leads to more aggressive trading by the insider on his information advantages regarding both the fundamental information and the noise supply (larger magnitudes of f_1 and f_2, see proposition 6). Importantly, this hypothesis serves as a direct test of the existence of the feedback effect.

Because institutional investors are more likely to be informed investors, we proxy the insider by institutional investors. In particular, we proxy insider's trading by the change in institutional ownership, as do Bennett, Sias, and Starks (2003), Yan and Zhang (2009), and Baik, Kang, and Kim (2010). Following Kallunki, Nilsson. and Hellstrom (2009), we proxy the fundamental information in the current period by earnings or returns in the next period, and proxy the noise supply by the aggregate holdings of individual investors.

4.2 Insider trading, inventory, and stock return

This subsection derives more empirical predictions by studying the relationships concerning insider's current trade, his previous inventory, and future stock returns. We discretize the processes of $Y=\left(I-\widehat{I},\quad U-\widehat{U},\quad X\right)$ and Q by sampling the time periods evenly. Recall that insider's optimal trading strategy is given by

$$dX=\left[f_1(I-\widehat{I})+f_2(U-\widehat{U})+f_3X\right]dt. \tag{45}$$

Using Proposition 2, we obtain that $I-\widehat{I}$, $U-\widehat{U}$, and X satisfy the differential equations:

$$\begin{pmatrix} I_{t+\tau}-\widehat{I}_{t+\tau} \\ U_{t+\tau}-\widehat{U}_{t+\tau} \\ X_{t+\tau} \end{pmatrix}=e^{A_Y\tau}\begin{pmatrix} I_t-\widehat{I}_t \\ U_t-\widehat{U}_t \\ X_t \end{pmatrix}+u_{t+\tau}, \tag{46}$$

where

$$A_Y = \begin{bmatrix} -\alpha_I(1+m_1)-h_1 f_1 & (a+f_3-f_2)h_1 & 0 \\ -h_2 f_1 - m_2\alpha_I & -a+(a+f_3-f_2)h_2 & 0 \\ f_1 & f_2 & f_3 \end{bmatrix},$$

$$u_{t+\tau} = \int_{s=0}^{\tau} e^{A_Y(\tau-s)} B_Y dB(t+s),$$

$$B_Y = \begin{bmatrix} -(\eta\sigma_D + m_1\sigma_D) & \sqrt{2\eta-\eta^2}\sigma_D & -h_1\sigma_U \\ -m_2\sigma_D & 0 & (1-h_2)\sigma_U \\ 0 & 0 & 0 \end{bmatrix}.$$

The insider's order flow at time t is defined as $X_t - X_{(t-\tau)}$. We then compute the excess dollar return $Q_t - Q_{t-\tau}$ of the stock from Equation (18) and the expression for Y in Proposition 2. Q_t has the following solution (see e.g. Arnold, 1974):

$$Q_{t+\tau} - Q_t = \psi^T (Y_{t+\tau} - Y_t) + v_{\{t,\tau\}}, \tag{47}$$

where the constant vector ψ satisfies $\psi^T(a_Y + \zeta f^T) = a_Q^T + \lambda f^T$ and $v_{\{t,\tau\}} = \int_t^{t+\tau} (b_Q^T - \psi^T b_Y) dB$. In the multivariate regression of

$$Q_{t+\tau} - Q_t = \beta_1 (X_t - X_{t-\tau}) + \beta_2 X_{t-\tau} + \epsilon_{t+\tau}, \tag{48}$$

the coefficients are given by

$$\begin{bmatrix} \beta_1(\tau) \\ \beta_2(\tau) \end{bmatrix} = \begin{bmatrix} Var(X_t - X_{t-\tau}) & Cov(X_t - X_{t-\tau}, X_{t-\tau}) \\ Cov(X_t - X_{t-\tau}, X_{t-\tau}) & Var(X_{t-\tau}) \end{bmatrix}^{-1} \times \begin{bmatrix} Cov(Q_{t+\tau} - Q_t, X_t - X_{t-\tau}) \\ Cov(Q_{t+\tau} - Q_t, X_{t-\tau}) \end{bmatrix}. \tag{49}$$

Panels A and B of Figure 3 plot $\beta_1(\tau)$ and $\beta_2(\tau)$ against holding-horizon τ for different mean-reverting speeds of the noise supply (a). This figure illustrates that both $\beta_1(\tau)$ and $\beta_2(\tau)$ are positive for different τ's; that is, both insider's trade at time t, $X_t - X_{t-\tau}$, and his inventory at time $t-\tau$, $X_{t-\tau}$, are related positively to future returns. Because the risk-averse insider trades against his inventory position, $X_t - X_{t-\tau}$ and $X_{t-\tau}$ are negatively correlated due to the hedging effect, which strengthens each other's explanatory power in the multivariate regression. As a result, the coefficients, $\beta_1(\tau)$ and $\beta_2(\tau)$, are larger than those obtained by regressing $Q_{t+\tau} - Q_t$ on $X_t - X_{t-\tau}$ and $X_{t-\tau}$ separately.[32]

Empirically, Bennett, Sias, and Starks (2003), Yan and Zhang (2009), and Baik, Kang, and Kim (2010) decompose the institutional ownership

[32] Mathematically, if dependent variable X_1 and independent variable X_2 follow normal distributions and if X_1 and X_2 are negatively related, then the above result can be easily verified.

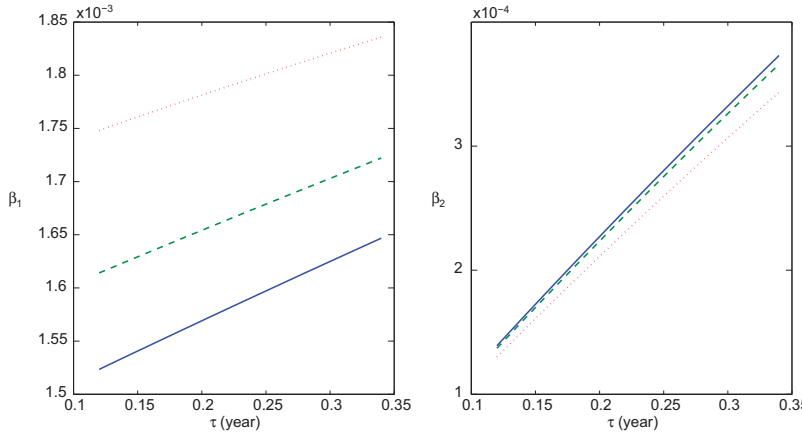

Figure 3
The forecasting power of insider's order in the current period and his inventory in the previous period
Panels A and B plot $\beta_1(\tau)$ and $\beta_2(\tau)$ against holding-horizon τ for different mean-reverting coefficients of noise supply (a), where $\beta_1(\tau)$ and $\beta_2(\tau)$ are estimated from Equation (49). We set $r=0.04$, $\sigma_D=0.2$, $\sigma_U=1500$, $k=0.001$, $\alpha_I=0.25$, and $\eta=0.1$. The solid, dashed, and dotted lines correspond to $a=1/12$, $a=1/4$, and $a=1/2$, respectively.

into the change in institutional ownership (CIO), which proxies for the private information on the stock payoff (fundamental information), and the lagged institutional ownership (LIO), which proxies for the demand shock (nonfundamental information). They find that both CIO and LIO are positively related to future stock returns.[33] Our model provides potential explanations for these empirical results, whereas other models, which are developed separately under fundamental information such as those by Kyle (1985) and CV (2008), or under nonfundamental information, such as that by Vayanos (2001), are inconsistent with them.[34]

Because our model focuses on the impact of noise traders' overreaction. We next derive additional new empirical implications regarding the impact of the correction speed of noise traders' overreaction, a, on the forecasting powers on future returns by insider's trade in the current period and his inventory in the previous period. Figure 3 illustrates that $\beta_1(\tau)$, the forecasting power on future stock returns by insider's trade in the current period, increases with a, whereas

[33] Gompers and Metrick (2001) also find that LIO is positively related to future stock returns, and that CIO is marginally significantly positive, with a t value of 1.75. The significance levels of CIO and LIO are fairly close.

[34] In strategic trading models without nonfundamental information, such as Kyle (1985) and CV (2008), the insider is risk neutral and his trade in the current period is given by $\beta(I-\widehat{I})$. His current trade and his previous inventory are positively correlated, because $(I-\widehat{I})$ follows a continuous-time AR(1) process. Because the current trade incorporates more recent information than the previous inventory, after controlling for the current trade, the inventory in the previous period does not forecast positively future stock returns. In inventory models such as Vayanos (2001), we expect the trade in the current period to forecast future returns negatively. Because of the risk-sharing motive, the insider demands liquidity, the current stock price is pushed up (down) by a buy (sell) order, which tends to be mean reverting to compensate the risk-averse market markers for providing immediacy service.

$\beta_2(\tau)$, the forecasting power on future stock returns by insider's holdings in the previous period, decreases with a. Intuitively, because noise trading pushes the stock price away from the fundamental value and the insider is risk averse, the insider hedges the noise risk. If the noise supply reverts to its mean faster, then the insider is less concerned about the noise risk. As a result, the hedging effect declines, and the insider trades less against his inventory, leading to a smaller $\beta_2(\tau)$. On the other hand, because of the feedback effect, the insider trades more aggressively on his fundamental information (in the spirit of Proposition 6), leading to a larger f_1 and a larger $\beta_1(\tau)$. In short, the empirical implication is that when the correction speed of noise traders' overreaction increases, the forecasting power on future stock returns by insider's trade in the current period increases, whereas the forecasting power by insider's holdings in the previous period declines.

5. Conclusion

This paper develops an infinite-horizon continuous-time model with a risk-averse insider and a mean-reverting noise supply or overreacting noise traders. The insider enjoys both fundamental (the expected growth rate of dividends) and nonfundamental (noise supply or insider's inventory) information advantages over market makers. A feedback effect arises from the insider's trading on nonfundamental information and fundamental information. Specifically, insider's trading on his nonfundamental information camouflages his private fundamental information, thus allowing the insider to trade more aggressively on his fundamental information. This effect is caused by the mean reversion of the noise supply or the risk aversion of the insider.

Because of the feedback effect, the equilibrium stock price is not fully revealing, although the insider may trade infinitely aggressively on his private fundamental information. This result holds even when the insider is risk neutral but the noise supply is mean reverting. In contrast, the price is fully revealing in previous strategic trading models in infinite-horizon, such as Vayanos (2001) and Chau and Vayanos (2008), where the feedback effect is absent. As the insider becomes more risk averse, the price impact decreases for a small amount of private fundamental information but increases for a large amount. When the feedback effect is sufficiently strong, a risk-averse insider can trade even more aggressively on his fundamental information than a risk-neutral one does. When the correction speed of noise traders' overreaction increases, the stock price in our model becomes more volatile, whereas that in Campbell and Kyle (1993) and Wang (1993) becomes less volatile in the absence of the feedback effect. Because of the feedback effect, an increase in the insider's risk aversion can also cause the price to be more volatile. Hence, the feedback effect is a mechanism that destabilizes the stock price.

Our model generates several unique empirical implications. We show that a faster correction speed of noise traders' overreaction leads to a higher

idiosyncratic volatility of the stock price, as well as to more aggressive trading by the insider on his information advantages regarding both the fundamental information and the noise supply. We demonstrate that both insider's order flow in the current period and his inventory in the previous period can forecast positively future stock returns, consistent with recent empirical evidence. We also illustrate that when the correction speed of noise traders' overreaction increases, the forecasting power of insider's trade in the current period on future stock returns increases, but the forecasting power of insider's holdings in previous periods declines.

For tractability, we assume that there is only one monopolistic insider. We believe, however, that most of our results, particularly the feedback effect, should remain qualitatively the same, even with multiple insiders. In this case, each insider would still trade on his own signal about the fundamental information, which is correlated across insiders. Insiders would still be better informed about the noise supply than would market makers. Hence, just like in the case of one insider, each insider would also trade on his information advantage regarding the noise supply, still allowing him to trade more aggressively on his fundamental information advantage. The nonfully revealing price should hold when the transaction cost goes to zero, because insiders would still trade on both types of information advantages. Considering multiple insiders would represent an interesting extension of this model, but the technical difficulty is that we would encounter the infinite regress problem when insiders' private signals are not perfectly correlated.

Appendix A: Derivations of the Fundamental Value and the Price Function

We derive the fundamental value in Equation (4) and the price function in Equation (8). The processes of $y=\begin{pmatrix} D \\ I \end{pmatrix}$ are given by

$$dy_t = a_y y_t dt + c_y dB_t, \tag{A1}$$

where $a_y=\begin{pmatrix} 0 & \alpha_I \\ 0 & -\alpha_I \end{pmatrix}$, $c_y=\begin{pmatrix} \sigma_D & 0 \\ -\eta\sigma_D & \sqrt{2\eta-\eta^2}\sigma_D \end{pmatrix}$, and $dB=\begin{pmatrix} dB_1 \\ dB_2 \end{pmatrix}$. y_t can be expressed in an integral form as

$$y_{t+s}=\beta_s y_t+\int_{\tau=0}^{s} e^{a_y(s-\tau)}c_y dB_{t+\tau},$$

where $\beta_s=e^{a_y s}=\begin{pmatrix} \beta_{11}(s) & \beta_{12}(s) \\ \beta_{21}(s) & \beta_{22}(s) \end{pmatrix}$. Because D_t cannot predict I_{t+s}, we must have $\beta_{21}(s)=0$ for $s\geq 0$.

Solving differential equation $d\beta_s/ds=a_y\beta_s$, with boundary condition $\beta_0=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, yields

$$\beta_{11}(s)=1,\quad \beta_{12}(s)=1-\exp(-\alpha_I s),\quad \beta_{22}(s)=\exp(-\alpha_I s).$$

Denote $\mathcal{F}(t)$ as the σ-field generated by $\{D_s, I_s : s\in(-\infty,t]\}$. We obtain

$$E[D_{t+s}|\mathcal{F}(t)]=\beta_{11}(s)D_t+\beta_{12}(s)I_t.$$

Because $E[D_t|\mathcal{F}_M(t)]=D_t$, we have

$$E[D_{t+s}|\mathcal{F}_M(t)]=\beta_{11}(s)D_t+\beta_{12}(s)E[I_t|\mathcal{F}_M(t)].$$

Thus, we obtain

$$P_t=E\left[\int_{s=0}^{+\infty}\exp(-rs)D_{t+s}ds|\mathcal{F}_M(t)\right]$$

$$=\int_{s=0}^{+\infty}\{\exp(-rs)D_t+\exp(-rs)\beta_{12}(s)E[I_t|\mathcal{F}_M(t)]\}ds=\frac{D_t}{r}+\mu\widehat{I}_t,$$

$$V_t=E\left[\int_0^{\infty}\exp(-rs)D_{t+s}ds|\mathcal{F}(t)\right]=\frac{D_t}{r}+\mu I_t,$$

where $\mu=\frac{\alpha_I}{r(r+\alpha_I)}$.

Appendix B: Proof of Proposition 1

We solve the filtering problem of market makers given insider's trading strategy. Suppose that the optimal order rate of the insider is given by

$$\theta=f^T y+g^T y_c, \tag{A2}$$

where $y=(I,U,X)^T$, $y_c=(\widehat{I},\widehat{U},\widehat{X})^T$, $f=(f_1,f_2,f_3)^T$, and $g=(-f_1,-f_2,0)^T$, with T denoting the transpose. Conjecture that y_c satisfies the following stochastic process:[35]

$$dy_c=a_c y_c dt+hd\omega+md D, \tag{A3}$$

where a_c is a 3×3 matrix, $h=(h_1,h_2,h_3)^T$, and $m=(m_1,m_2,m_3)^T$. Using Equation (A2), the process of y can be written in the following form:

$$dy=(a_0 y+e_0 y_c)dt+\sigma_y^T dB,$$

where $a_0=\begin{pmatrix}-\alpha_I & 0 & 0\\ 0 & -a & 0\\ f_1 & f_2 & f_3\end{pmatrix}$, $e_0=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -f_1 & -f_2 & 0\end{pmatrix}$, $\sigma_y=\begin{pmatrix}-\eta\sigma_D & 0 & 0\\ \sqrt{2\eta-\eta^2}\sigma_D & 0 & 0\\ 0 & \sigma_U & 0\end{pmatrix}$, and $dB=\begin{pmatrix}dB_1\\ dB_2\\ dB_3\end{pmatrix}$.

Applying Ito's lemma to $\omega=X+U$ yields

$$d\omega=(f^T y+g^T y_c-aU)dt+\begin{pmatrix}0, & 1, & 0\end{pmatrix}\sigma_y^T dB.$$

By the rearrangement of Equation (A3), we have

$$dy_c=(a_1 y+e_1 y_c)dt+\sigma_{y,c}dB,$$

where $a_1=hf^T-ah(0,1,0)+\alpha_I m(1,0,0)$, $e_1=hg^T+a_c$, $\sigma_{yc}=\begin{pmatrix}m\sigma_D,\bar{0},h\sigma_U\end{pmatrix}$ is a 3×3 matrix, and $\bar{0}=(0,0,0)^T$. Because y_c is observable, observing $d\omega$ is equivalent to observing

$$d\varsigma_1=(f^T y-aU)dt+\begin{pmatrix}0, & 1, & 0\end{pmatrix}\sigma_y^T dB.$$

[35] By verification, we shall show that y_c indeed follows such a process.

Let $\varsigma=(\varsigma_1, D)^T$. Stack y and y_c together and let $Z_Y=\begin{pmatrix} y \\ y_c \end{pmatrix}$. Conditional on the observation of ς, the posterior mean of Z_Y, denoted by $\widehat{Z}_Y=\begin{pmatrix} y_c \\ y_c \end{pmatrix}$, is given by[36]

$$dy_c=(a_0+e_0)y_c dt+\left[\Sigma q_0^T+q_{1zs}\right]q_{ss}^{-1}d\overline{B}, \tag{A4}$$

$$dy_c=\left[(a_c+hg^T+hf^T)y_c-ah\widehat{U}+\alpha_I m\widehat{I}\right]dt+q_{2zs}q_{ss}^{-1}d\overline{B}, \tag{A5}$$

where $q_0=\begin{pmatrix} f_1 & f_2-a & f_3 \\ \alpha_I & 0 & 0 \end{pmatrix}$, $q_{ss}=\begin{pmatrix} \sigma_U^2 & 0 \\ 0 & \sigma_D^2 \end{pmatrix}$, $q_{1zs}=\begin{pmatrix} 0 & -\eta\sigma_D^2 \\ \sigma_U^2 & 0 \\ 0 & 0 \end{pmatrix}$, $q_{2zs}=\begin{pmatrix} h_1\sigma_U^2 & m_1\sigma_D^2 \\ h_2\sigma_U^2 & m_2\sigma_D^2 \\ h_3\sigma_U^2 & m_3\sigma_D^2 \end{pmatrix}$, and $d\overline{B}=d\varsigma-\begin{pmatrix} f^T y_c-a\widehat{U} \\ \alpha_I\widehat{I} \end{pmatrix}dt$. Comparing the drift and diffusion of y_c in Equations (A4) and (A5) yields

$$\begin{aligned} 0 &= \Sigma q_0^T+q_{1zs}-q_{2zs}, \\ 0 &= a_c+hg^T+hf^T-ah(0,1,0)+\alpha_I m(1,0,0)-a_0-e_0. \end{aligned}$$

Rearranging these equations, we have

$$a_c=\begin{bmatrix} -\alpha_I(1+m_1) & ah_1 & -f_3h_1 \\ -m_2\alpha_I & -a(1-h_2) & -f_3h_2 \\ -m_3\alpha_I & ah_3 & f_3(1-h_3) \end{bmatrix} \tag{A6}$$

and

$$\begin{pmatrix} h_1\sigma_U^2 & m_1\sigma_D^2 \\ h_2\sigma_U^2 & m_2\sigma_D^2 \\ h_3\sigma_U^2 & m_3\sigma_D^2 \end{pmatrix}=\Sigma q_0^T+\begin{pmatrix} 0 & -\eta\sigma_D^2 \\ \sigma_U^2 & 0 \\ 0 & 0 \end{pmatrix}. \tag{A7}$$

Let $O_t=E_t\left[(Z_{Y_t}-\widehat{Z}_{Y_t})(Z_{Y_t}-\widehat{Z}_{Y_t})^T|\mathcal{F}_M\right]$ denote the variance-covariance matrix. By calculation, it is given by $O_t=\begin{pmatrix} \Sigma_t & 0 \\ 0 & 0 \end{pmatrix}$. We consider only a steady-state Bayesian-Nash equilibrium, focusing on the steady-state solution $dO_t=0$. We next prove that Σ is given by

$$0=a_0\Sigma+\Sigma a_0^T+\Omega-\left[\Sigma q_0^T+q_{1zs}\right](q_{ss})^{-1}[\Sigma q_0+q_{1zs}]^T,$$

where $\Omega=\begin{pmatrix} 2\eta\sigma_D^2 & 0 & 0 \\ 0 & \sigma_U^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Following Liptser and Shiryaev (2001), the steady-state solution for $dO_t=0$ is given by

$$\begin{aligned} 0=O\begin{pmatrix} a_0 & e_0 \\ a_1 & e_1 \end{pmatrix}^T+\begin{pmatrix} a_0 & e_0 \\ a_1 & e_1 \end{pmatrix}O+\begin{pmatrix} \Omega & \Omega_1 \\ \Omega_1^T & \Omega_2 \end{pmatrix} \\ -\left[O\begin{pmatrix} q_0^T \\ 0 \end{pmatrix}+\begin{pmatrix} q_{1zs} \\ q_{2zs} \end{pmatrix}\right](q_{ss})^{-1}\left[O\begin{pmatrix} q_0^T \\ 0 \end{pmatrix}+\begin{pmatrix} q_{1zs} \\ q_{2zs} \end{pmatrix}\right]^T, \end{aligned}$$

[36] See Lipster and Shiryaev (2001) or Appendix A of Wang (1993).

where $\Omega_1 = \sigma_y^T \sigma_{yc}^T$ and $\Omega_2 = \sigma_{y_c} \sigma_{y_c}^T = mm^T \sigma_D^2 + hh^T \sigma_U^2$. Using partitioned matrices, we have

$$\Sigma a_0^T + a_0 \Sigma + \Omega = \left[\Sigma q_0^T + q_{1zs}\right](q_{ss})^{-1}\left[\Sigma q_0^T + q_{1zs}\right]^T, \tag{A8}$$

$$0 = a_1 \Sigma + \Omega_1^T - [q_{2zs}](q_{ss})^{-1}\left[\Sigma q_0^T + q_{1zs}\right]^T, \tag{A9}$$

$$0 = \Omega_2 - (q_{2zs})(q_{ss})^{-1} q_{2zs}^T. \tag{A10}$$

We first verify that Equations (A9) and (A10) hold. By matrix manipulation, it can be shown that $q_{2zs}(q_{ss})^{-1} q_{2zs} = \sigma_{y_c} \sigma_{y_c}^T = mm^T \sigma_D^2 + hh^T \sigma_U^2 = \Omega_2$. Hence, Equation (A10) holds. We know that $a_1 \Sigma = (h,m) q_0 \Sigma$, $\Omega_1^T = (h,m) q_{1zs}^T$, and $[q_{2zs}](q_{ss})^{-1}\left[\Sigma q_0^T + q_{1zs}\right]^T = [q_{2zs}](q_{ss})^{-1}[q_{2zs}]^T = (h,m) q_{2zs}^T$. Equation (A7) means that $q_0 \Sigma + q_{1zs}^T - q_{2zs}^T = 0$. Therefore, Equation (A9) holds. As a result, Σ satisfies:

$$\begin{aligned} 0 = d\Sigma_t &= a_0 \Sigma + \Sigma a_0{}^T + \Omega - \left[\Sigma q_0^T + q_{1zs}\right](q_{ss})^{-1}\left[\Sigma q_0^T + q_{1zs}\right]^T, \\ &= a_0 \Sigma + \Sigma a_0{}^T + \Omega - q_{2zs}(q_{ss})^{-1} q_{2zs}, \\ &= a_0 \Sigma + \Sigma a_0{}^T + \Omega - mm^T \sigma_D^2 - hh^T \sigma_U^2. \end{aligned}$$

From Lemma 1, we obtain that $X - \widehat{X} = -U + \widehat{U}$, leading to the solution

$$\Sigma = \Lambda^T \bar{\Sigma} \Lambda,$$

where $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ and $\bar{\Sigma}$ is given by

$$0 = \bar{a}_0 \bar{\Sigma} + \bar{\Sigma} \bar{a}_0^T + \bar{\Omega} - \left[\bar{\Sigma} q_0^T + q_1\right](q_s)^{-1}\left[\bar{\Sigma} q_0^T + q_1\right]^T, \tag{A11}$$

where $\bar{\Omega} = \begin{pmatrix} 2\eta\sigma_D^2 & 0 \\ 0 & \sigma_U^2 \end{pmatrix}$, $q_0^T = \begin{pmatrix} f_1 & \alpha_I \\ f_2 - a - f_3 & 0 \end{pmatrix}$, $q_1 = \begin{pmatrix} 0 & -\eta\sigma_D^2 \\ \sigma_U^2 & 0 \end{pmatrix}$, $q_s = \begin{pmatrix} \sigma_U^2 & 0 \\ 0 & \sigma_D^2 \end{pmatrix}$, and $\bar{a}_0 = \begin{pmatrix} -\alpha_I & 0 \\ 0 & -a \end{pmatrix}$.

Plugging Equation (A7) into Equation (A11) yields

$$0 = \bar{a}_0 \bar{\Sigma} + \bar{\Sigma} \bar{a}_0^T + \bar{\Omega} - \bar{m}\bar{m}^T \sigma_D^2 - \bar{h}\bar{h}^T \sigma_U^2,$$

where $\bar{h} = (h_1, h_2)^T$ and $\bar{m} = (m_1, m_2)^T$. By rearrangement, $\bar{\Sigma} \equiv \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ is given by

$$2\alpha_I \Sigma_{11} = \left[2\eta\sigma_D^2 - h_1^2 \sigma_U^2 - m_1^2 \sigma_D^2\right], \quad (a + \alpha_I)\Sigma_{12} = -\left[h_1 h_2 \sigma_U^2 + m_1 m_2 \sigma_D^2\right],$$

$$2a\Sigma_{22} = \left[(1 - h_2^2)\sigma_U^2 - m_2^2 \sigma_D^2\right].$$

As $d\widehat{U} + d\widehat{X} = d\omega$, using Equation (A3), we obtain

$$1 = h_2 + h_3, \quad 0 = m_2 + m_3.$$

Appendix C: Proof of Proposition 3

We solve the optimization problem of the insider. Proposition 2 gives the expression for dY. Conjecture that the value function of the insider is of the form

$$J(W,Y,t)=-\frac{1}{r\gamma}\exp\left(-\rho t-r\gamma W-\Phi_0-\frac{1}{2}Y^T LY\right), \tag{A12}$$

where L is a 3×3 symmetric matrix and Φ_0 is a constant. By calculation, the Bellman equation in Equation (20) reduces to

$$0=\max_{C,\theta}-\frac{1}{\gamma}\exp(-\rho t-\gamma C)+J_t+\frac{1}{2}tr(J_{YY}b_Y b_Y^T)+J_W(rW-C-\frac{1}{2}k\theta^2+X\lambda\theta+Xa_Q^T Y)$$

$$+J_Y^T(a_Y Y+\zeta\theta)+\frac{1}{2}J_{WW}X^2 b_Q^T b_Q+J_{YW}^T Xb_Y b_Q, \tag{A13}$$

where $J_W=-r\gamma J$, $J_t=-\rho J$, $J_Y=-LYJ$, $J_{WW}=(r\gamma)^2 J$, $J_{YY}=(-L+LYY^T L)J$, and $J_{YW}=r\gamma LYJ$. a_Q, b_Q, a_Y, ζ, and b_Y are defined in Proposition 2.

The first-order condition (FOC) with respect to the consumption rate C yields

$$-J_W+\exp[-\rho t-\gamma C]=0. \tag{A14}$$

By rearrangement, we obtain

$$C=rW+\frac{1}{\gamma}\left(\Phi_0+\frac{1}{2}Y^T LY\right). \tag{A15}$$

The FOC with respect to the order rate θ yields

$$-JY^T L\zeta-r\gamma J(\lambda X-k\theta)=0$$

Rearrangement gives

$$\theta=Y^T\frac{L\zeta+r\gamma\lambda i_3}{r\gamma k},\quad i_3=\begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{A16}$$

Substituting the optimal C and θ into the Bellman Equation (A13) yields

$$0=(r-\rho)-\frac{1}{2}tr(b_Y^T Lb_Y)+\left[r(\Phi_0+\frac{1}{2}Y^T LY)-\frac{1}{2}r\gamma k\theta^2-r\gamma Xa_Q^T Y\right]$$

$$+\frac{1}{2}(Y^T Lb_Y b_Y^T LY)-Y^T La_Y Y+\frac{1}{2}(r\gamma)^2X^2 b_Q^T b_Q+r\gamma XY^T Lb_Y b_Q,$$

where tr denotes the trace of a matrix. Note that this equation is the sum of two terms. The first is independent of Y and the second is a quadratic function of Y. A solution to this problem is that these two terms are both zero. We then have

$$(r-p)+r\Phi_0-\frac{1}{2}tr(b_Y^T Lb_Y)=0, \tag{A17}$$

$$0=Y^T\left[\frac{1}{2}rL-r\gamma i_3 a_Q^T+\frac{1}{2}(r\gamma)^2(i_3 i_3^T)(b_Q^T b_Q)-La_Y+\frac{1}{2}Lb_Y b_Y^T L+r\gamma Lb_Y b_Q i_3^T\right]Y$$

$$-\frac{1}{2r\gamma k}Y^T[(L\zeta+r\gamma\lambda i_3)(L\zeta+r\gamma\lambda i_3)]Y. \tag{A18}$$

Solving Equation (A17) yields Φ_0:

$$\Phi_0 = -\frac{(r-\rho)}{r} + \frac{1}{2r} tr(b_Y^T L b_Y). \tag{A19}$$

Because the right-hand side of Equation (A18) is a scalar, it is equal to half of the sum of this expression and its transpose. L then satisfies the following equation:

$$0 = rL - r\gamma(i_3 a_Q^T) - r\gamma(a_Q i_3^T) - (a_Y^T L + L a_Y) + L b_Y b_Y^T L + (r\gamma)^2 (b_Q^T b_Q)(i_3 i_3{}^T)$$

$$- r\gamma k \left(\frac{L\zeta}{r\gamma k} + \frac{\lambda i_3}{k}\right)\left(\frac{L\zeta}{r\gamma k} + \frac{\lambda i_3}{k}\right)^T + (r\gamma) L b_Y b_Q i_3{}^T + (r\gamma) i_3 b_Q^T b_Y^T L. \tag{A20}$$

Simple calculation yields the results in Proposition 3.

C.1: The optimality of the solution to the Bellman equation

The last step is to show that the solution to the Bellman equation is optimal. Following Back (2010), we require two technical conditions on the solution to the Bellman equation:

$$E_t\left[N(W_s, Y_s, s)^2 Y_s^T \bar{G} Y_s\right] < \infty, \tag{A21}$$

$$\lim_{s\to\infty} E_t[J(W_s, Y_s, s)] = 0, \tag{A22}$$

where $N(W,Y,t) = -\exp(\rho t) J(W,Y,t)$, $E_t(.)$ denotes the conditional expectation based on the insider's information set at time t, and $\bar{G} = (r\gamma)^2 (i_3 i_3^T)(b_Q^T b_Q) + L b_Y b_Y^T L + r\gamma i_3 b_Q^T b_Y^T L + r\gamma L b_Y b_Q i_3^T$.

We first prove Equation (A21). Because Y follows a multidimensional Ornstein-Uhlenbeck process, Y_s is normally distributed conditional on Y_t, where $Y_t^T = (I_t - \widehat{I}_t, U_t - \widehat{U}_t, X_t)$. Hence, $E_t\{[Y_s^T \bar{G} Y_s]^2\} < \infty$ is satisfied. Plugging the expression for C in Equation (A15) into the wealth process in Equation (20) yields

$$dW = (a_M^T X Y - V_0 - \frac{Z_0}{\gamma} - \frac{k}{2}\theta^2 - \frac{1}{2\gamma} Y^T L Y) dt + X b_Q dB, \tag{A23}$$

where $a_M = \left[\frac{\alpha_I}{r} + \mu m_1 \alpha_I + \lambda f_1, \quad -\lambda(a + f_3 - f_2), \quad 0\right]^T$. From Proposition 2, we know that $dY = A_Y Y dt + b_Y dB$, where

$$A_Y = \begin{bmatrix} -\alpha_I(1+m_1) - h_1 f_1 & (a + f_3 - f_2) h_1 & 0 \\ -h_2 f_1 - m_2 \alpha_I & -a + (a + f_3 - f_2) h_2 & 0 \\ f_1 & f_2 & f_3 \end{bmatrix},$$

and $b_Y = \begin{bmatrix} -(\eta\sigma_D + m_1\sigma_D) & \sqrt{2\eta - \eta^2}\sigma_D & -h_1\sigma_U \\ -m_2\sigma_D & 0 & (1-h_2)\sigma_U \\ 0 & 0 & 0 \end{bmatrix}$. Substituting the above expressions into $N(W_t, Y_t, t)$ and applying Ito's lemma gives

$$N(W_s, Y_s, s) = A_1 \exp\left(-r\gamma W_t - \frac{1}{2} Y_t^T L Y_t - \int_t^s Y_z^T \bar{F}_1 Y_z dz - \int_t^s Y_z^T \bar{F}_2 dB_z\right)$$

$$= A_1 \exp\left(-r\gamma W_t - \frac{1}{2} Y_t^T L Y_t - \int_t^s Y_z^T \bar{F}_1 Y_z dz\right) \times \exp\left(-\int_t^s Y_z^T \bar{F}_2 dB_z\right),$$

where $s > t$, $A_1 = \frac{1}{r\gamma} \exp[-Z_0 - \gamma V_0 - (r-\rho)(s-t)]$, $\bar{F}_1 = L A_Y + r\gamma[(0,0,1)^T a_M^T - \frac{1}{2\gamma} L - \frac{k}{2} f f^T]$, $\bar{F}_2 = r\gamma(0,0,1)^T b_Q + L b_Y$, and $f^T = (f_1, \quad f_2, \quad f_3)$.

By Hölder's inequality, we obtain

$$E_t|N(W_s,Y_s,s)| \le A_1 \exp[-r\gamma W_t - Y_t^T L Y_t] \times \left[E_t \exp(-2\int_t^s Y_z^T \bar{F}_2 dB_z)\right]^{1/2}$$

$$\times E_t\left[\exp(-2\int_t^s Y_z^T \bar{F}_1 Y_z dz)\right]^{1/2}.$$

Because $\int_t^s -2Y_z^T \bar{F}_2 dB_z$ is a martingale and Y_s follows a normal distribution conditional on the information set at time t, we obtain

$$E_t \exp\left[-2\int_t^s Y_z^T \bar{F}_2 dB_z\right] < \infty, \quad E_t \exp\left[-2\int_t^s Y_z^T \bar{F}_1 Y_z dz\right] < \infty, \quad E_t\{[Y_s^T \bar{G} Y_s]^2\} < \infty.$$

Therefore, $E_t[N(W_s,Y_s,s)] < \infty$ and $E_t[N(W_s,Y_s,s)^4] < \infty$. Hölder's inequality then ensures that

$$E_t[N(W_s,Y_s,s)^2 Y_s^T \bar{G} Y_s] \le \{E_t[N(W_s,Y_s,s)^4] \times E_t\{[Y_s^T \bar{G} Y_s]^2\}\}^{1/2} < \infty.$$

We next show that $\lim_{s\to\infty} E_t[J(W_s,Y_s,s)] = 0$. By rearrangement, Equation (A14) yields

$$-\frac{1}{\gamma}\exp[-\rho s - \gamma C_s] = rJ(W,Y,s).$$

The Bellman equation is then given by

$$0 = E_s[rJ(W_s,Y_s,s)dt + dJ(W_s,Y_s,s)].$$

Taking expectation at time t yields

$$0 = E_t[rJ(W_s,Y_s,s)dt + dJ(W_s,Y_s,s)].$$

By Equation (A21), rearranging the above equation gives

$$E_t[J(W_s,Y_s,s)] = \exp[-r(s-t)]E_t[J(W_t,Y_t,t)]. \tag{A24}$$

Taking the limit of $s \to \infty$ yields

$$\lim_{s\to\infty} E_t[J(W_s,Y_s,s)] = 0.$$

Given the two conditions in Equations (A21) and (A22), we now proceed to demonstrate that the solution to the Bellman equation gives an upper bound on the value function of the optimization problem (19). We prove the verification theorem, that is, the consumption rate C_t and the position in the stock X_t attaining the maximum in the Bellman equation are optimal. In every state at time s, the value function $J(W_t,Y_t,t)$ satisfies

$$J(W_s,Y_s,s) = J(W_t,Y_t,t) + \int_t^s dJ, \qquad s \ge t.$$

Calculating the insider's expected utility yields

$$E_t\left[-\int_t^\infty \frac{1}{\gamma}\exp(-\rho s - \gamma C_s)ds\right]$$

$$= E_t\left[-\int_t^{t_0} \frac{1}{\gamma}\exp(-\rho s - \gamma C_s)ds\right] + J(W_t,Y_t,t) + E_t\left[\int_t^{t_0} dJ(W_s,Y_s,s)\right]$$

$$= J(W_t,Y_t,t) + E_t\left\{\int_t^{t_0} \bar{H}_s ds - [r\gamma X_s b_Q^T + Y_s^T L b_Y] J_s dB_s\right\}, \tag{A25}$$

where $\bar{H} = -\frac{1}{\gamma}\exp(-\rho s - \gamma C_s) + J_t + \frac{1}{2}tr(J_{YY} b_Y b_Y^T) + J_W(rW - C - \frac{1}{2}k\theta^2 + X\lambda\theta + Xa_Q^T Y) + J_Y^T(a_Y Y + \zeta\theta) + \frac{1}{2}J_{WW}X^2 b_Q^T b_Q + J_{YW}^T X b_Y b_Q$.

Because $N(W_s, Y_s, s)$ is a continuous function, Equation (A21) implies that there exists an A_0 so that $E_t\left[N(W_s, Y_s, s)^2 Y_s^T \bar{G} Y_s\right] < A_0$ for any $s \in [t, t_0]$. Taking integral from t to t_0 and using Equation (A21) yields

$$E_t\left[\int_t^{t_0} J^2 Y^T \bar{G} Y ds\right] \le \int_t^{t_0} \exp(-2\rho s) A_0 ds \le \frac{A_0}{2\rho}[\exp(-2\rho t_0) - \exp(-2\rho t)] < \infty.$$

It implies that the stochastic integral

$$\int_t^{t_0} [r\gamma X_s b_Q^T + Y_s^T L b_Y] J_s dB_s$$

is a martingale. Therefore, $E_t\{\int_t^{t_0}[r\gamma X_s b_Q^T + Y_s^T L b_Y] J_s dB_s\} = 0$ and Equation (A25) reduces to

$$E_t\left[-\int_t^{t_0} \frac{1}{\gamma}\exp(-\rho s - \gamma C_s) ds\right] + E_t\left[J(t_0, W_{t_0}, Y_{t_0})\right] = J(W_t, Y_t, t) + E_t\left\{\int_t^{t_0} \bar{H}_s ds\right\}.$$

Note that the integrand on the right-hand side has a maximum value of zero according to the Bellman equation. For a general consumption rate C and the optimal position in the stock X, the expression must be negative or zero everywhere in the integral, meaning that

$$E_t\left[-\int_t^{t_0} \frac{1}{\gamma}\exp(-\rho s - \gamma C_s) ds\right] + E_t\left[J(W_{t_0}, Y_{t_0}, t_0)\right] \le J(t, W_t, Y_t).$$

Taking the limit of $t_0 \to \infty$ and using the monotone convergence theorem and condition (A22), we obtain

$$E_t\left[-\int_t^{\infty} \frac{1}{\gamma}\exp(-\rho s - \gamma C_s) dt\right] \le J(W_t, Y_t, t),$$

for any admissible C and X, and the equality holds for C^* and X^* attained in the Bellman equation.

Appendix D: Proof of Corollary 2

We first conjecture the value function and then solve for the FOCs and the Bellman equation by letting $\gamma = 0$ and $\rho = r$ in the corresponding equations in the proof of Proposition 3. We use the same notations as in Proposition 3. As γ goes to zero, $1/\gamma \exp(-\rho t - \gamma C)$ converges to infinity. Hence, we interpret the insider's utility function as $1/\gamma \exp(-\rho t)[\exp(-\gamma C) - 1]$. Using L'Hôspital's rule, we can rewrite Equations (A12), (A15), (A16), and (A20), which give the optimal solution of the insider. His value function reduces to

$$J(W, Y, t) = \frac{1}{r}\exp(-rt)\left(rW + \bar{V}_0 + 0.5 Y^T \bar{L} Y\right), \tag{A26}$$

where $\bar{L} = L/\gamma$ is a 3×3 symmetric matrix, $\bar{V}_0$ is a constant, and L is defined in Proposition 3.

The FOC with respect to insider's order rate θ yields:

$$\theta = Y^T \frac{\bar{L}\zeta + r\lambda i_3}{rk}, \quad i_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The Bellman equation then reduces to

$$0 = r\bar{L} - r(i_3 a_Q^T) - r(a_Q i_3^T) - (a_Y^T \bar{L} + \bar{L} a_Y) - rk\left(\frac{\bar{L}\zeta}{rk} + \frac{\lambda i_3}{k}\right)\left(\frac{\bar{L}\zeta}{rk} + \frac{\lambda i_3}{k}\right)^T,$$

and $\bar{V}_0$ is given by

$$\bar{V}_0 = \frac{1}{2r} tr(b_Y^T \bar{L} b_Y).$$

Plugging the expression for $\bar{V}_0$ into Equation (A15) yields

$$C = rW + \frac{1}{2} Y^T \bar{L} Y + \frac{1}{2r} tr(b_Y^T \bar{L} b_Y).$$

Appendix E: Proof of Proposition 4

Using the excess return process dQ in equation (18), we have

$$\sigma_P^2 = b_Q^T Cov(dB, dB^T) b_Q = \left[\frac{\sigma_D}{r} + \mu m_1 \sigma_D\right]^2 + [\lambda \sigma_U]^2.$$

Substituting the expression for Σ_{11} into Equation (16) yields

$$\sigma_P^2 = \frac{\sigma_D^2}{r^2} + 2\mu^2(\eta\sigma_D^2 - \Sigma_{11}\alpha_I) + \frac{2\sigma_D\mu}{r}(m_1\sigma_D).$$

Appendix F: Proof of Proposition 5

We study the limiting behavior of the results in Propositions 1 through 3. We prove by "conjecture and verify" that the economy converges to the one of fully revealing; that is, we first assume that the economy converges to the one of fully revealing and then verify that the results in Propositions 1 through 3 are satisfied.

We assume that as $k \to 0$,

$$\frac{\bar{\Sigma}_{11}}{\sqrt{k}} \to \widetilde{\Sigma_{11}}, \quad \sqrt{k} f_1 \to \widetilde{f_1}, \quad f_2 = f_3 = 0. \tag{A27}$$

We first examine market makers' filtering problem. As the insider trades only on $I - \widehat{I}$, we solve market makers' updating belief regarding the fundamental information. Assume that h_1 and m_1 converge to $\widetilde{h_1}$ and $\widetilde{m_1}$ as $k \to 0$. Equations (15) and (16) give the expressions for h, m, and Σ. Substituting Equation (A27) into Equation (12) yields

$$\widetilde{m_1} \to -\eta.$$

Because Equations (12) and (16) hold when $k \to 0$, we obtain

$$\widetilde{h_1} \to \sqrt{2\eta - \eta^2}\frac{\sigma_D}{\sigma_U}, \quad \widetilde{\Sigma_{11}}\widetilde{f_1} - \widetilde{h_1}\sigma_U^2 \to 0. \tag{A28}$$

Rearranging Equation (A28) yields

$$\widetilde{\Sigma_{11}} \to \frac{h_1\sigma_U^2}{\widetilde{f_1}}.$$

After solving market makers' filtering problem, we next solve the insider's problem. We assume that $\bar{L} \to \widetilde{L} = \begin{pmatrix} \widetilde{L_{11}} & \widetilde{L_{12}} & \widetilde{L_{13}} \\ \widetilde{L_{21}} & \widetilde{L_{22}} & \widetilde{L_{23}} \\ \widetilde{L_{31}} & \widetilde{L_{32}} & \widetilde{L_{33}} \end{pmatrix}$ as $k \to 0$, where $\widetilde{L}$ is a symmetric matrix. Plugging the relevant parameters into the Y process defined in Proposition 2, we find that as $k \to 0$, Y defined in Equation (17) converges to the following process.

$$d(I - \widehat{I}) = -(1-\eta)\alpha_I(I - \widehat{I})dt - h_1\theta dt + \sqrt{2\eta - \eta^2}\sigma_D dB_2 - h_1\sigma_U dB_3,$$

and the excess return process dQ given in Equation (18) converges to

$$dQ = dP + (D - rP)dt = a_Q^T Y dt + \lambda\theta dt + b_Q^T dB,$$

where a_Q and b_Q are given by

$$a_Q = \left(\tfrac{\alpha_I}{r} - \eta\mu\alpha_I, \quad 0, \quad 0\right)^T, \quad b_Q = \left(\tfrac{\sigma_D}{r} - \mu\eta\sigma_D, \quad 0, \quad \mu\sqrt{2\eta - \eta^2}\sigma_D\right)^T.$$

As $k \to 0$, the limit of insider's order rate given in (27) satisfies the following equation:

$$\sqrt{k}\theta = \frac{r\mu\widetilde{h}_1 X + Y^T\widetilde{L}\widetilde{\zeta}}{r\sqrt{k}} \to Y^T\begin{pmatrix}\widetilde{f}_1\\0\\0\end{pmatrix}, \quad \widetilde{\zeta} = \begin{pmatrix}-\widetilde{h}_1\\-\widetilde{h}_2\\1\end{pmatrix}. \tag{A29}$$

Equation (29) reduces to

$$r\widetilde{L_{22}} \to 0, \quad \left[\frac{r}{2} + (1-\eta)\alpha_I + \frac{r}{2}\right]\widetilde{L_{12}} \to 0, \quad \widetilde{L_{23}} = \widetilde{L_{33}} \to 0,$$

$$2\left[\frac{r}{2} + (1-\eta)\alpha_I\right]\widetilde{L_{11}} - r\widetilde{f}_1^{\,2} \to 0, \quad \widetilde{L_{13}} - \frac{a_s r}{r + (1-\eta)\alpha_I} \to 0,$$

where $a_s = \frac{\alpha_I}{r} - \eta\mu\alpha_I$. Rearranging the above expressions yields

$$\widetilde{L} \to \begin{pmatrix}\frac{r\widetilde{f}_1^{\,2}}{r+2(1-\eta)\alpha_I} & 0 & r\mu\\ 0 & 0 & 0\\ r\mu & 0 & 0\end{pmatrix}.$$

Equation (A29) requires that

$$-\widetilde{L_{11}}\widetilde{h}_1 + \widetilde{L_{13}} \to 0, \tag{A30}$$

$$r\mu\widetilde{h}_1 - \widetilde{L_{31}}\widetilde{h}_1 \to 0, \quad -\widetilde{L_{12}}\widetilde{h}_1 - \widetilde{L_{22}}\widetilde{h}_2 + \widetilde{L_{23}} \to 0. \tag{A31}$$

Simple calculations show that Equation (A31) holds. Rearranging Equation (A30) yields

$$\widetilde{f}_1 \to \sqrt{\frac{\mu[r + 2(1-\eta)\alpha_I]}{\widetilde{h}_1}}.$$

It can be shown that the certainty equivalent annualized steady-state value of private information is given by

$$\bar{V}_0 \to \frac{1}{2r} tr(b_Y^T \widetilde{L} b_Y) = \mu\sqrt{(2\eta - \eta^2)}\sigma_D\sigma_U.$$

Therefore, we have shown that as $k \to 0$, the price tends to be fully revealing, but the value of private information is positive.

Appendix G: Proof of Proposition 6

We study the behavior of the results in Propositions 1 through 3 when k and a are small, by letting k and a converge to zero and imposing the condition that $\gamma = 0$. Notice that the equilibrium in this case is driven only by the feedback effect. Our proofs are divided into two steps. First, we prove by contradiction that the equilibrium does not converge to a fully revealing one as k goes to zero for any $a > 0$. Second, we prove by "conjecture and verify" that the equilibrium converges to a nonfully revealing one.

G.1: Proof by contradiction

Conjecture that we still have a fully revealing equilibrium as $k \to 0$ for any $a > 0$, which implies that $\Sigma_{11} \to 0$. Plugging this expression into Equations (12) and (16) yields

$$m_1 \to -\eta, \quad h_1 \to \sqrt{(2\eta - \eta^2)}\frac{\sigma_D}{\sigma_U}.$$

When $a > 0$, Σ_{22} is a finite number, as $\Sigma_{22} \leq \frac{\sigma_U^2}{2a}$. From Hölder's inequality, we obtain that

$$\Sigma_{12} \leq {\Sigma_{11}}^{1/2} {\Sigma_{22}}^{1/2} \to 0. \tag{A32}$$

Using Equation (12), we obtain

$$m_2 \to 0, \quad m_3 \to 0.$$

Using Equation (16) and $h_2 + h_3 = 1$, we further have

$$h_2 \to 0, \quad h_3 \to 1, \quad 2a\Sigma_{22} \to \sigma_U^2.$$

We also obtain from Proposition 1 that

$$\begin{cases} h_1\sigma_U^2 = \Sigma_{11} f_1 + \Sigma_{12}(f_2 - a) \\ h_2\sigma_U^2 = \Sigma_{12} f_1 + \Sigma_{22}(f_2 - a). \end{cases} \tag{A33}$$

Note that the above equations imply that f_1 and f_2 cannot be finite. As the insider is risk neutral, $f_3 = 0$.

From Proposition 2, we obtain that $a_Q = [a_s, -a\lambda, 0]$, where $a_s = \frac{\alpha_I}{r} - \alpha_I \eta \mu$ and

$$a_Y \to \begin{pmatrix} -(1-\eta)\alpha_I & ah_1 & 0 \\ 0 & -a & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Corollary 2 simplifies to

$$r\begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} - r\begin{pmatrix} 0 & 0 & a_s \\ 0 & 0 & -a\lambda \\ a_s & -a\lambda & 0 \end{pmatrix} - rk\begin{pmatrix} f_1^2 & f_1 f_2 & 0 \\ f_1 f_2 & f_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$- \begin{pmatrix} -2(1-\eta)\alpha_I L_{11} & ah_1 L_{11} - aL_{12} - (1-\eta)\alpha_I L_{12} & -(1-\eta)\alpha_I L_{13} \\ ah_1 L_{11} - aL_{12} - (1-\eta)\alpha_I L_{12} & 2(ah_1 L_{12} - aL_{22}) & ah_1 L_{13} - aL_{23} \\ -(1-\eta)\alpha_I L_{13} & ah_1 L_{13} - aL_{23} & 0 \end{pmatrix} \to 0.$$

We thus obtain $L_{33} \to 0$. Similarly, we have

$$rL_{13} - ra_s + (1-\eta)\alpha_I L_{13} \to 0, \quad rL_{23} + ar\lambda - (ah_1 L_{13} - aL_{23}) \to 0,$$

$$rL_{11} - rkf_1^2 + 2(1-\eta)\alpha_I L_{11} \to 0.$$

Simplification yields

$$L_{13} \to r\mu, \quad L_{23} \to 0, \quad L_{11} \to \frac{rkf_1^2}{r + 2(1-\eta)\alpha_I}.$$

Plugging them into Equation (27) yields that $f_3 = \frac{(-h_1 L_{31} + L_{33}) + r\lambda}{rk} \to 0$. We also obtain

$$rL_{12} - rkf_1 f_2 - (ah_1 L_{11} - aL_{12}) + (1-\eta)\alpha_I L_{12} \to 0, \tag{A34}$$

$$rL_{22} - rkf_2^2 - 2a(h_1 L_{12} - L_{22}) \to 0, \tag{A35}$$

$$-h_1 L_{12} + L_{23} \to rkf_2, \tag{A36}$$

$$-h_1 L_{11} + L_{13} \to rkf_1. \tag{A37}$$

Substituting Equation (A37) into the expression for L_{11} yields

$$(r\mu - h_1 L_{11})^2 \to rk[rL_{11} + 2(1-\eta)\alpha_I L_{11}].$$

As $k \to 0$, we obtain $r\mu - h_1 L_{11} \to 0$. Rearrangement yields $L_{11} \to \frac{r\mu}{h_1}$. Thus, $f_1 \to \sqrt{[r + 2(1-\eta)\alpha_I]\frac{\mu}{h_1 k}}$, which is proportional to $1/\sqrt{k}$.

Plugging Equation (A36) into Equation (A34) and applying $L_{23} \to 0$ yields

$$rL_{12} - f_1(-h_1 L_{12}) - (ah_1 L_{11} - aL_{12}) + (1-\eta)\alpha_I L_{12} \to 0.$$

Rearrangement gives $[r + a + (1-\eta)\alpha_I + h_1 f_1]L_{12} \to ah_1 L_{11} \to ar\mu$ or $L_{12} \to \frac{ar\mu}{[r+a+(1-\eta)\alpha_I + h_1 f_1]}$. Using Equation (A36), we then have

$$f_2 \to \frac{-h_1}{rk} L_{12} = \frac{-a\lambda}{k[r + a + (1-\eta)\alpha_I + h_1 f_1]},$$

which is proportional to $1/\sqrt{k}$ when k is small. Plugging the expressions for f_1 and f_2 into Equation (A33) yields

$$\Sigma_{11} f_1 + \Sigma_{12} f_2 \to \sqrt{(2\eta - \eta^2)}\sigma_U \sigma_D, \tag{A38}$$

$$\Sigma_{12} f_1 + \frac{\sigma_U^2}{2a} f_2 \to \frac{\sigma_U^2}{2}. \tag{A39}$$

From Equation (A39), we obtain that $\Sigma_{12} \to \frac{\sigma_U^2}{2f_1} - \frac{\sigma_U^2 f_2}{2af_1}$ as k goes to zero. Because f_2/f_1 goes to a nonzero constant, we know that Σ_{12} converges to a nonzero constant, which is contradicted with Equation (A32). Hence, the equilibrium cannot be fully revealing.

G.2: Proof by "conjecture and verify"

We further use the expansion techniques to determine the equilibrium as $k \to 0$ and $a \to 0$. We conjecture that

$$\Sigma_{11} \to \sqrt{k}\widetilde{\Sigma_{11}} + a\underline{\Sigma_{11}}, \quad \Sigma_{12} \to \underline{\Sigma_{12}}, \quad \Sigma_{22} \to \frac{\widetilde{\Sigma_{22}}}{a},$$

$$f_1 \to \frac{\widetilde{f_1} + a\underline{f_1}}{\sqrt{k}}, \quad f_2 \to \frac{a\underline{f_2}}{\sqrt{k}}, \quad f_3 = 0,$$

and $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$ and $\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$ converge to $\begin{pmatrix} \widetilde{h_1} + a\underline{h_1} \\ \widetilde{h_2} + a\underline{h_2} \\ \widetilde{h_3} + a\underline{h_3} \end{pmatrix}$ and $\begin{pmatrix} \widetilde{m_1} + a\underline{m_1} \\ \widetilde{m_2} + a\underline{m_2} \\ \widetilde{m_3} + a\underline{m_3} \end{pmatrix}$, respectively. We also conjecture that $\bar{L} \to (\widetilde{L} + a\underline{L}) = \begin{pmatrix} \widetilde{L_{11}} + a\underline{L_{11}} & \widetilde{L_{12}} + a\underline{L_{12}} & \widetilde{L_{13}} + a\underline{L_{13}} \\ \widetilde{L_{21}} + a\underline{L_{21}} & \widetilde{L_{22}} + a\underline{L_{22}} & \widetilde{L_{23}} + a\underline{L_{23}} \\ \widetilde{L_{31}} + a\underline{L_{31}} & \widetilde{L_{32}} + a\underline{L_{32}} & \widetilde{L_{33}} + a\underline{L_{33}} \end{pmatrix}$, where $\widetilde{L}$ and $\underline{L}$ are both symmetric matrices and $\bar{L}$ is defined in Corollary 2.

We first examine market makers' filtering problem. We let both k and a go to zero. Fully revealing private information gives

$$\widetilde{h_1} \to \sqrt{2\eta - \eta^2}\frac{\sigma_D}{\sigma_U}, \quad \widetilde{m_1} \to -\eta, \quad \widetilde{\Sigma_{11}} \to \frac{h_1 \sigma_U^2}{\widetilde{f_1}}.$$

We then consider the limiting case of k going to zero with $a > 0$. Plugging the conjectured expressions for $\bar{\Sigma}$, h, and m into Proposition 1, we obtain

$$\underline{\Sigma_{11}}\widetilde{f_1} + \underline{\Sigma_{12}}\underline{f_2} \to 0, \quad \underline{\Sigma_{12}}\widetilde{f_1} + \widetilde{\Sigma_{22}}\underline{f_2} \to 0, \tag{A40}$$

$$\alpha_I \underline{\Sigma_{12}} \to -(\sigma_U^2 \widetilde{h_1}\widetilde{h_2} + \sigma_D^2 \widetilde{m_1}\widetilde{m_2}), \quad \widetilde{m_2}\sigma_D^2 \to \alpha_I \underline{\Sigma_{12}}, \quad \underline{m_1}\sigma_D^2 \to \alpha_I \underline{\Sigma_{11}}, \tag{A41}$$

$$\widetilde{h_1}\underline{h_1}\sigma_U^2 + \widetilde{m_1}\underline{m_1}\sigma_D^2 + \alpha_I \underline{\Sigma_{11}} \to 0, \quad 2\widetilde{\Sigma_{22}} \to (1 - \widetilde{h_2}^2)\sigma_U^2 - \widetilde{m_2}^2\sigma_D^2. \tag{A42}$$

Therefore, we obtain

$$\frac{\underline{\Sigma_{11}}}{\underline{\Sigma_{12}}} = \frac{\underline{\Sigma_{12}}}{\widetilde{\Sigma_{22}}} \to -\frac{\underline{f_2}}{\widetilde{f_1}}, \tag{A43}$$

$$\underline{h_1} \to -\frac{(1-\eta)\underline{m_1}\sigma_D^2}{\widetilde{h_1}\sigma_U^2}. \tag{A44}$$

Letting both a and k go to zero in Equations (27) and (29), we obtain

$$\widetilde{L_{33}} = 0, \quad \widetilde{L_{32}} \to 0, \quad \widetilde{L_{13}} \to r\mu, \quad \widetilde{L_{12}} \to 0,$$

$$\widetilde{L_{22}} \to 0, \quad \widetilde{L_{11}} \to \frac{r\mu}{\widetilde{h_1}}, \quad \widetilde{f_1} \to \sqrt{\frac{[r + 2(1+\widetilde{m_1})\alpha_I]\widetilde{L_{11}}}{r}}.$$

Therefore, the value function $\bar{L}$ converges to the one derived in Proposition 5, in which a and γ are equal to zero and k goes to zero. To make economic sense, $\widetilde{f_1} > 0$. Hence, we require $r + 2(1-\eta)\alpha_I > 0$.

After solving market makers' filtering problem, we solve the insider's problem. Given the expressions for f_1, f_2, and f_3 in Equation (27), we obtain

$$\underline{L_{33}} - \widetilde{h_1}\underline{L_{13}} - \widetilde{h_2}\underline{L_{23}} \to 0, \quad \underline{L_{23}} - \widetilde{h_1}\underline{L_{12}} - \widetilde{h_2}\underline{L_{22}} \to 0,$$

$$\underline{L_{13}} - \widetilde{h_1}\underline{L_{11}} - \widetilde{h_2}\underline{L_{12}} - \widetilde{L_{11}}\underline{h_1} \to 0. \tag{A45}$$

Plugging Equation (A45) into Equation (29) and comparing the order of a on both sides of Equation (29) gives

$$\underline{L_{12}} \to 0, \quad \underline{L_{13}} \to 0, \quad \underline{L_{22}} \to 0, \quad \underline{L_{23}} \to 0, \quad \underline{L_{33}} \to 0,$$

$$r\mu + r\widetilde{f_1}\underline{f_2} \to 0, \quad [r + 2(1-\eta)\alpha_I]\underline{L_{11}} + 2\underline{m_1}\alpha_I\widetilde{L_{11}} \to r2\widetilde{f_1}\underline{f_1}.$$

Simplification yields

$$\underline{f_2} \to -\frac{\mu}{\widetilde{f_1}} < 0, \quad \underline{f_1} \to \frac{[r + 2(1-\eta)\alpha_I]\underline{L_{11}} + 2\underline{m_1}\alpha_I\widetilde{L_{11}}}{2r\widetilde{f_1}}. \tag{A46}$$

Equation (A45) reduces to

$$\underline{L_{11}} \to -\frac{h_1}{\widetilde{h_1}}\widetilde{L_{11}}.$$

We next determine $\widetilde{h_2}$ and $\widetilde{m_2}$. Plugging the expression for $\underline{f_2}$ into Equations (A42) and (A43) yields

$$\frac{2\widetilde{m_2}\sigma_D^2}{\alpha_I[(1-\widetilde{h_2}^2)\sigma_U^2-\widetilde{m_2}^2\sigma_D^2]}\to\frac{\mu}{\widetilde{f_1}^2}, \tag{A47}$$

$$\widetilde{m_2}\to-\frac{\widetilde{h_1}\widetilde{h_2}\sigma_U^2}{(1+\widetilde{m_1})\sigma_D^2},\quad \text{if } \eta\neq 1,\quad \widetilde{h_2}\to 0,\quad \text{if } \eta=1. \tag{A48}$$

Because $\underline{\Sigma_{22}}>0$, from Equation (A43), we know that $\underline{\Sigma_{12}}>0$. Using Equation (A41), we obtain that $\widetilde{m_2}>0$. Rearranging Equations (A47) and (A48) yields

$$\widetilde{m_2}=\frac{-(\sigma_D\widetilde{f_1}^2)+\sqrt{\sigma_D^2\widetilde{f_1}^4+\alpha_I^2\mu^2\sigma_U^2}}{\alpha_I\mu\sigma_D},\quad \text{if } \eta=1,$$

$$b_1\widetilde{h_2}^2+b_2\widetilde{h_2}+b_3\equiv-\frac{\mu}{\widetilde{f_1}^2}[\frac{\widetilde{h_1}^2\sigma_U^2}{(1+\widetilde{m_1})^2\sigma_D^2}+1]\widetilde{h_2}^2+\frac{2\widetilde{h_1}}{\alpha_I(1+\widetilde{m_1})}\widetilde{h_2}+\frac{\mu}{\widetilde{f_1}^2}\to 0,\quad \text{if } \eta\neq 1.$$

When $\eta\neq 1$, because $b_1b_3<0$, we know that there exists a unique solution to the above quadratic function, which satisfies that $\widetilde{h_2}<0$ ($\eta<1$) or $\widetilde{h_2}>0$ ($\eta>1$). Rearranging Equations (A43) and (A44) yields $\underline{\Sigma_{11}}>0$ and $\underline{m_1}>0$, respectively. Therefore, we have shown that there exists a nonfully revealing equilibrium when $a>0$ even though k goes to zero. Note that if $\eta<1$, then $\underline{h_1}<0$; if $\eta=1$, then $\underline{h_1}=0$; and if $\eta>1$, then $\underline{h_1}>0$. Hence, the effect of the mean version of the noise supply on the price impact depends on the amount of private information.

We next prove that $\underline{f_1}>0$. When $\eta\leq 1$, because $\underline{L_{11}}\geq 0$ and $\widetilde{L_{11}}\geq 0$, Equation (A46) yields that $\underline{f_1}>0$. When $\eta>1$, we assume that $r(\eta-1)<2\alpha_I$. Rearranging Equation (A46) and using equation (A44) yields $\underline{f_1}\to\frac{\alpha_I\underline{m_1}\widetilde{L_{11}}}{r\widetilde{f_1}}\left(1-\frac{[r+2(1-\eta)\alpha_I](\eta-1)}{2\alpha_I(2\eta-\eta^2)}\right)>0$.

Applying Proposition 4, the instantaneous variance rate of the price σ_p^2 is given by

$$\sigma_P^2\to\sigma_{p_f}^2+\left(\frac{2\mu\sigma_D^2\underline{m_1}}{r+\alpha_I}\right)a,$$

where $\sigma_{p_f}^2=\frac{\sigma_D^2}{r^2}-2\mu^2\frac{r}{\alpha_I}\eta\sigma_D^2\leq\sigma_{P_D}^2$ is the fully revealing price variability. Because $\underline{m_1}>0$, a rise in a leads to a rise in σ_P^2. As the price deviates more from the fundamental value, it is more volatile.

Appendix H: Proof of Proposition 7

We consider the case in which the insider is risk averse. We study the limiting behavior of the results in Propositions 1 through 3 by letting k and γ converge to zero. To obtain results compatible with those in CV (2008), we impose the condition that $a=0$. We prove by "conjecture and verify" that the equilibrium converges to a nonfully revealing one. Compared with Section 3.2, the equilibrium is more complicated and driven by the feedback effect and the hedging effect, both of which are induced by insider's risk aversion. We assume that the economy does not converge to the one of fully revealing and then verify that the results in Propositions 1 through 3 are satisfied. We conjecture that f_1, f_2, and f_3 are all nonzero.

We use the expansion techniques to determine the equilibrium as $k\to 0$ and $\gamma\to 0$. We conjecture that

$$\Sigma_{11}\to\sqrt{k}\widetilde{\Sigma_{11}}+\gamma\underline{\Sigma_{11}},\quad \Sigma_{12}\to\underline{\Sigma_{12}},\quad \Sigma_{22}\to\frac{\widetilde{\Sigma_{22}}}{\gamma},$$

$$f_1\to\frac{\widetilde{f_1}+\gamma\underline{f_1}}{\sqrt{k}},\quad f_2\to\frac{\gamma\underline{f_2}}{\sqrt{k}},\quad f_3\to\gamma\underline{f_3},$$

and $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$ and $\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$ converge to $\begin{pmatrix} \widetilde{h_1}+\gamma \underline{h_1} \\ \widetilde{h_2}+\gamma \underline{h_2} \\ \widetilde{h_3}+\gamma \underline{h_3} \end{pmatrix}$ and $\begin{pmatrix} \widetilde{m_1}+\gamma \underline{m_1} \\ \widetilde{m_2}+\gamma \underline{m_2} \\ \widetilde{m_3}+\gamma \underline{m_3} \end{pmatrix}$. We also conjecture that $\bar{L} \to (\widetilde{L}+\gamma \underline{L}) = \begin{pmatrix} \widetilde{L_{11}}+\gamma \underline{L_{11}} & \widetilde{L_{12}}+\gamma \underline{L_{12}} & \widetilde{L_{13}}+\gamma \underline{L_{13}} \\ \widetilde{L_{21}}+\gamma \underline{L_{21}} & \widetilde{L_{22}}+\gamma \underline{L_{22}} & \widetilde{L_{23}}+\gamma \underline{L_{23}} \\ \widetilde{L_{31}}+\gamma \underline{L_{31}} & \widetilde{L_{32}}+\gamma \underline{L_{32}} & \widetilde{L_{33}}+\gamma \underline{L_{33}} \end{pmatrix}$, where $\widetilde{L}$ and $\underline{L}$ are both symmetric matrices and $\bar{L} \equiv L/\gamma$.

We first examine market makers' filtering problem. We let both k and γ go to zero. Fully revealing private information gives

$$\widetilde{h_1} \to \sqrt{2\eta-\eta^2}\frac{\sigma_D}{\sigma_U}, \quad \widetilde{m_1} \to -\eta, \quad \widetilde{\Sigma_{11}} \to \frac{h_1 \sigma_U^2}{\widetilde{f_1}}.$$

We then consider the limiting case of k going to zero with a positive γ. Plugging the conjectured expressions for $\bar{\Sigma}$, h, and m into Proposition 1 yields

$$\underline{\Sigma_{11}}\widetilde{f_1}+\underline{\Sigma_{12}}\underline{f_2} \to 0, \quad \underline{\Sigma_{12}}\widetilde{f_1}+\widetilde{\Sigma_{22}}\underline{f_2} \to 0, \tag{A49}$$

$$\begin{aligned} &\alpha_I \underline{\Sigma_{12}} \to -(\sigma_U^2 \widetilde{h_1}\widetilde{h_2}+\sigma_D^2 \widetilde{m_1}\widetilde{m_2}), \quad \widetilde{h_1}\underline{h_1}\sigma_U^2+\widetilde{m_1}\underline{m_1}\sigma_D^2+\alpha_I \underline{\Sigma_{11}} \to 0, \\ &\underline{m_1}\sigma_D^2 \to \alpha_I \underline{\Sigma_{11}}, \quad \widetilde{m_2}\sigma_D^2 \to \alpha_I \underline{\Sigma_{12}}, \quad (1-\widetilde{h_2}^2)\sigma_U^2-\sigma_D^2 \widetilde{m_2}^2 \to 0. \end{aligned} \tag{A50}$$

Rearranging Equation (A49) yields

$$\frac{\underline{\Sigma_{11}}}{\underline{\Sigma_{12}}} = \frac{\underline{\Sigma_{12}}}{\widetilde{\Sigma_{22}}} \to -\frac{\underline{f_2}}{\widetilde{f_1}}. \tag{A51}$$

From Equation (A50), we have

$$\widetilde{m_2} \to \frac{\alpha_I}{\sigma_D^2}\underline{\Sigma_{12}}, \quad \widetilde{h_2} \to -\frac{(1-\eta)\alpha_I}{\sqrt{2\eta-\eta^2}\sigma_D \sigma_U}\underline{\Sigma_{12}}, \qquad |\underline{\Sigma_{12}}| \to \frac{\sqrt{(2\eta-\eta^2)}\sigma_D \sigma_U}{\alpha_I}.$$

Using Equation (A51), we know that if $\underline{f_2} > 0$, then $\underline{\Sigma_{12}} \to -\frac{\sqrt{(2\eta-\eta^2)}\sigma_D \sigma_U}{\alpha_I}$ and that if $\underline{f_2} < 0$, then $\underline{\Sigma_{12}} \to \frac{\sqrt{(2\eta-\eta^2)}\sigma_D \sigma_U}{\alpha_I}$.

After solving market makers' filtering problem, we solve the insider's problem. Plugging the relevant parameters into the Y process defined in Equation (17) yields

$$\begin{aligned} dY &\to (\widetilde{a_Y}+\gamma \underline{a_Y})Y dt+(\widetilde{\zeta}+\gamma \underline{\zeta})\theta dt+(\widetilde{b_Y}+\gamma \underline{b_Y})dB \\ &\to \begin{bmatrix} -(1-\eta+\gamma \underline{m_1})\alpha_I & \gamma \underline{f_3}\widetilde{h_1} & \gamma \underline{f_3}\widetilde{h_1} \\ -\alpha_I(\widetilde{m_2}+\gamma \underline{m_2}) & \gamma \underline{f_3}\widetilde{h_2} & \gamma \underline{f_3}\widetilde{h_2} \\ 0 & 0 & 0 \end{bmatrix} Y dt+\begin{bmatrix} -\widetilde{h_1}-\gamma \underline{h_1} \\ -\widetilde{h_2}-\gamma \underline{h_2} \\ 1 \end{bmatrix}\theta dt \\ &\quad +\begin{bmatrix} -\gamma \underline{m_1}\sigma_D & \sqrt{2\eta-\eta^2}\sigma_D & -(\widetilde{h_1}+\gamma \underline{h_1})\sigma_U \\ -(\widetilde{m_2}+\gamma \underline{m_2})\sigma_D & 0 & (1-\widetilde{h_2}-\gamma \underline{h_2})\sigma_U \\ 0 & 0 & 0 \end{bmatrix} dB, \end{aligned}$$

where we omit the terms with the order of γ higher than one.

The excess return process dQ given in Equation (18) converges to

$$dQ \to dP+(D-rP)dt = (\widetilde{a_Q^T}+\gamma \underline{a_Q^T})Y dt+\mu(\widetilde{h_1}+\gamma \underline{h_1})\theta dt+(\widetilde{b_Q^T}+\gamma \underline{b_Q^T})dB,$$

where $\widetilde{a_Q}$, $\widetilde{b_Q}$, $\underline{a_Q}$, and $\underline{b_Q}$ are given by

$$\widetilde{a_Q} \to \left[\tfrac{\alpha_I}{r} - \eta\mu\alpha_I, \quad 0, \quad 0\right]^T, \quad \underline{a_Q} \to \left[\mu\alpha_I \underline{m_1}, \quad -\mu \underline{f_3}\widetilde{h_1}, \quad -\mu \underline{f_3}\widetilde{h_1}\right]^T,$$

$$\widetilde{b_Q} \to \left(\tfrac{\sigma_D}{r} - \mu\eta\sigma_D, \quad 0, \quad \mu\sigma_U\widetilde{h_1}\right)^T, \quad \underline{b_Q} \to \left(\mu\sigma_D \underline{m_1}, \quad 0, \quad \mu\sigma_U \underline{h_1}\right)^T.$$

As $k \to 0$ and $\gamma \to 0$, the limit of the insider's order rate given in Equation (22) satisfies the following equation:

$$\sqrt{k}\theta \to \frac{r\mu(\widetilde{h_1} + \gamma \underline{h_1})X + Y^T(\widetilde{L} + \gamma \underline{L})(\widetilde{\zeta} + \gamma \underline{\zeta})}{r\sqrt{k}} \to Y^T \begin{pmatrix} \widetilde{f_1} + \gamma \underline{f_1} \\ \gamma \underline{f_2} \\ \sqrt{k}\gamma \underline{f_3} \end{pmatrix}, \tag{A52}$$

where $\widetilde{\zeta} = \begin{pmatrix} -\widetilde{h_1} \\ -\widetilde{h_2} \\ 1 \end{pmatrix}$ and $\underline{\zeta} = \begin{pmatrix} -\underline{h_1} \\ -\underline{h_2} \\ 0 \end{pmatrix}$.

Substituting the conjectured expressions into Proposition 3, letting k go to zero, and comparing the coefficients in front of γ on both sides of Equation (24), we obtain

$$\widetilde{L_{33}} \to 0, \quad \widetilde{L_{32}} \to 0, \quad \widetilde{L_{13}} \to r\mu, \quad \widetilde{L_{21}} \to 0,$$

$$\widetilde{L_{22}} \to 0, \quad \widetilde{L_{11}} \to \frac{r\mu}{\widetilde{h_1}}, \quad \widetilde{f_1} \to \sqrt{\frac{[r + 2(1 + \widetilde{m_1})\alpha_I]\mu}{\widetilde{h_1}}}.$$

Therefore, the value function parameter $\bar{L}$ converges to the one derived in Proposition 4, in which γ and a are equal to zero and k goes to zero.

We next derive the parameters $\underline{L}$, $\underline{f_1}$, $\underline{f_2}$, and $\underline{f_3}$. Given the expressions for f_1, f_2 and f_3 in Equation (A52), we obtain

$$\underline{L_{33}} - \widetilde{h_1}\underline{L_{13}} - \widetilde{h_2}\underline{L_{23}} \to 0, \quad \underline{L_{23}} - \widetilde{h_1}\underline{L_{12}} - \widetilde{h_2}\underline{L_{22}} \to 0,$$

$$\underline{L_{13}} - \widetilde{h_1}\underline{L_{11}} - \widetilde{h_2}\underline{L_{12}} - \widetilde{L_{11}}\underline{h_1} \to 0. \tag{A53}$$

Comparing the coefficients of γ^2 on both sides of Equation (24), we obtain

$$0 = r\underline{L} - r(i_3 \underline{a_Q}^T) - r(\underline{a_Q} i_3^T) - (\widetilde{a_Y}^T \underline{L} + \underline{L}\widetilde{a_Y} + \underline{a_Y}^T \widetilde{L} + \widetilde{L}\underline{a_Y})$$

$$- r[\widetilde{f}\,\underline{f}^T + \underline{f}\,\widetilde{f}^T] + \widetilde{L}\widetilde{b_Y}\widetilde{b_Y}^T\widetilde{L} + r^2(\widetilde{b_Q}^T\widetilde{b_Q})(i_3 i_3^T) + r\widetilde{L}\widetilde{b_Y}\underline{b_Q} i_3^T + r i_3 \underline{b_Q}^T \widetilde{b_Y}^T \widetilde{L}, \tag{A54}$$

where $\widetilde{f} = \begin{pmatrix} \widetilde{f_1} \\ 0 \\ 0 \end{pmatrix}$, $\underline{f} = \begin{pmatrix} \underline{f_1} \\ \underline{f_2} \\ 0 \end{pmatrix}$. Compared with the case in which the insider is risk neutral and the noise supply follows a random walk, the Bellman equation involves an extra term related to risk aversion, $\widetilde{L}\widetilde{b_Y}\widetilde{b_Y}^T\widetilde{L} + r^2(\widetilde{b_Q}^T\widetilde{b_Q})(i_3 i_3^T) + r\widetilde{L}\widetilde{b_Y}\underline{b_Q} i_3^T + r i_3 \underline{b_Q}^T \widetilde{b_Y}^T \widetilde{L}$, reflecting the hedging effect.

Simplification yields

$$\underline{L_{23}} \to 0, \quad \underline{L_{22}} \to 0, \quad \underline{L_{12}} \to 0, \quad \underline{L_{13}} \to \frac{\underline{L_{33}}}{\widetilde{h_1}},$$

$$\underline{L_{33}} \to -\frac{\sigma_D^2}{r}[1 + 2r\mu\eta(r\mu - 1)], \quad \underline{f_3} \to \frac{(r + \alpha_I - \alpha_I\eta)}{r\mu\widetilde{h_1}}\underline{L_{33}} + r\mu\widetilde{h_1}\sigma_U^2, \quad \underline{f_2} \to -\frac{\mu \underline{f_3}}{\widetilde{f_1}}.$$

Because $\mu = \frac{\alpha_I}{r(\alpha_I + r)}$, simple calculation shows that $r\mu < 1$. Hence, $2r\mu\eta(r\mu - 1) > -1$ and $\underline{L_{33}} < 0$. To make economic sense, we consider only the case in which $\underline{f_3} < 0$. Otherwise, the equilibrium

will not be a stationary one. Because $\underline{f_2} \to -\frac{\mu \underline{f_3}}{\widetilde{f_1}}$, we obtain that $\underline{f_2} > 0$. Given the expression for $\underline{f_2}$, rearranging Equations (A50) and (A51) yields

$$\underline{\Sigma_{11}} \to -\frac{\underline{f_2}\Sigma_{12}}{\widetilde{f_1}} \to \frac{\sqrt{2\eta-\eta^2}\sigma_D\sigma_U|\underline{f_2}|}{\alpha_I \widetilde{f_1}} > 0,$$

$$\underline{h_1} \to -\frac{(1-\eta)\alpha_I}{\sqrt{2\eta-\eta^2}\sigma_U\sigma_D}\underline{\Sigma_{11}} \to -(1-\eta)\frac{|\underline{f_2}|}{\widetilde{f_1}}.$$

Note that $\underline{h_1} < 0$ if $\eta < 1$ and $\underline{h_1} > 0$ if $\eta > 1$. Hence, $\lambda = \mu h_1$ is a decreasing function of γ when $\eta < 1$, but an increasing function of γ when $\eta > 1$. From Equation (A50), we obtain that $\underline{m_1} > 0$. Rearranging Equation (A53) yields $\underline{L_{11}} \to \frac{\underline{L_{33}} - r\mu\underline{h_1}}{\widetilde{h_1}^2}$. Using Equation (A54), we obtain $\underline{f_1}$, which is given by

$$\underline{f_1} \to \frac{\underline{L_{11}}[r+2\alpha_I(1-\eta)]+2r^2\mu^2\sigma_U^2 - \frac{2r\mu\alpha_I\underline{m_1}}{\widetilde{h_1}}}{2r\widetilde{f_1}}.$$

As the expressions for $\underline{f_1}$ and $\underline{f_2}$ are complicated, we are unable to determine their signs for the general case, even though we can obtain closed-form solutions. However, our numerical calculations show that when $\eta < 1$, the hedging effect dominates so that $\underline{f_1} < 0$ and $\underline{f_2} > 0$. Hence, as γ increases, the insider trades less aggressively on $I - \widehat{I}$ but more positively on $U - \widehat{U}$. In contrast, when $\eta > 1$, the feedback effect is more important so that $\underline{f_1} > 0$ and $\underline{f_2} > 0$.

Note that when k goes to zero, the price is still not fully revealing. Therefore, we have shown that there exists a nonfully revealing equilibrium when $\gamma > 0$ and k goes to zero. We calculate the annualized steady-state value of private information as follows:

$$\bar{V_0} = \frac{1}{2r}tr(b_Y^T \bar{L} b_Y) \to \frac{1}{2r}(\widetilde{L_{11}} + \gamma \underline{L_{11}})[(\widetilde{h_1} + \gamma \underline{h_1})^2\sigma_U^2 + (2\eta-\eta^2)\sigma_D^2 + (\eta + \widetilde{m_1} + \gamma \underline{m_1})^2\sigma_D^2].$$

A simple calculation shows that

$$\bar{V_0} \to \widetilde{V_0} + \gamma \underline{V_0},$$

where $\widetilde{V_0} = \mu\sqrt{2\eta-\eta^2}\sigma_D\sigma_U$ and $\underline{V_0} = \frac{\underline{h_1}\widetilde{h_1}\widetilde{L_{11}} + \widetilde{h_1}^2\underline{L_{11}}}{r}\sigma_U^2 \to \frac{\underline{L_{33}}\sigma_U^2}{r} < 0$. Hence, risk aversion reduces insider's expected utility.

Applying Proposition 4, the instantaneous variance rate of the price, σ_P^2, is given by

$$\sigma_P^2 \to \sigma_{P_f}^2 + \left(\frac{2\mu\sigma_D^2\underline{m_1}}{r+\alpha_I}\right)\gamma,$$

where $\sigma_{P_f}^2 = \frac{\sigma_D^2}{r^2} - 2\mu^2\frac{r}{\alpha_I}\eta\sigma_D^2 \le \sigma_{P_D}^2$ is the fully revealing price variability. Because $\underline{m_1} > 0$, a rise in γ leads to a rise in σ_P^2, and as a result, less private information is incorporated into the price.

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